Spring 2015, Math 111
Lab 4: Kinematics of Linear Motion

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February 24, 2015

Learning Objectives

Today, we will be looking at applications of derivatives in the field of kinematics.

Learning Objectives:

- Interpret the first derivative of a position function
- Interpret the first derivative of a velocity function
- Relate the sign of velocity and position of an objection

Kinematics of Linear Motion

Definition

Kinematics refers to the aspects of motion that arise from using calculus to connect the concepts of *position (e.g. displacement)*, *velocity* and *acceleration*.

Kinematics of Linear Motion is the study of particles, objects, or systems undergoing linear motion.

Position

Position is the location of an object with respect to its origin at a specific point in time.

Its units are in length (e.g. meters, feet)

Position is a vector. It has both magnitude and direction (i.e. 10 meters West of origin).

Displacement

Displacement is the resultant distance from a change in position.

$$\text{displacement} = \text{position}_1 - \text{position}_2$$

Its units are in length (e.g. meters, feet)

It is a vector and has both magnitude and direction (i.e. point *b* is 5 meters west of point *a*).

Velocity

Velocity is the rate of change of position.

$$\text{velocity} = \frac{\text{change in position}}{\text{change in time}}$$

It is in units of length verus time (e.g. $m/s, ft/s$).

Velocity is a vector. It has both magnitude and direction (i.e. $10m/s$ North).

Acceleration

Acceleration is the rate of change of distance.

$$\text{acceleration} = \frac{\text{distance}}{\text{change in time}}$$

It is in units of length verus time (e.g. $m/s, ft/s$).

Acceleration is a scalar. It has only magnitude, no direction (i.e. $10m/s$).

Velocity versus Acceleration

It can be difficult to understand the relationship between *velocity* and *acceleration*.

	Velocity Positive (+)	Velocity Negative (-)
Acceleration Positive (+)	Object $\rightarrow$ "speeding up"	Object $\leftarrow$ "slowing down"
Acceleration Negative (-)	Object $\rightarrow$ "slowing down"	Object $\leftarrow$ "speeding up"

We can use this table to help us grasp how these two concepts interact.

Kinematic Equations

The following table summarizes the relationship between the three concepts.

Concept	**Equations**		
Position	$s(t)$		
Velocity	$v(t)$	$s'(t)$	
Acceleration	$a(t)$	$v'(t)$	$s''(t)$

Given a position equation, you can utilize the derivative concept to find both the velocity and acceleration equations.

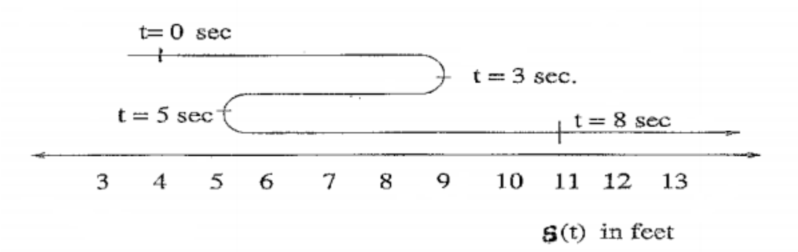
Types of Motion

In this lab, we will study two types of motion.

- **Straight Motion** in one dimension.
  - Uniform velocity
  - Uniform acceleration
- **Projectile Motion** in two dimensions.
  - Vertical component
  - Horizontal component

Straight Motion

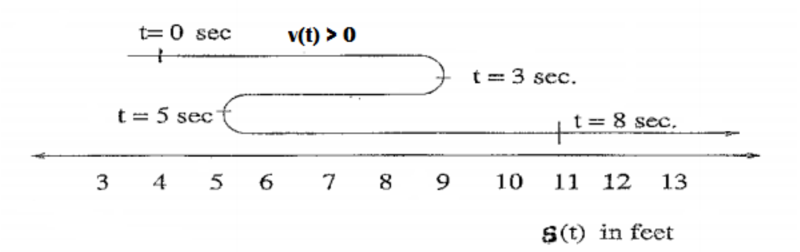
The following path diagram is used to visualize the path of an object on a straight line path (an object with straight motion).



The object is moving in one dimension (sliding back and forth). We visualize this by 'stacking' the movement of the object in the diagram.

Straight Motion

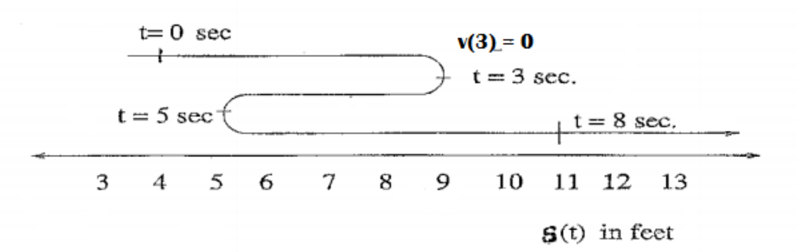
We see that from $t = 0$ to $t = 3$, the object is moving to the right of the starting position.



Therefore, the velocity is positive in this interval.

Straight Motion

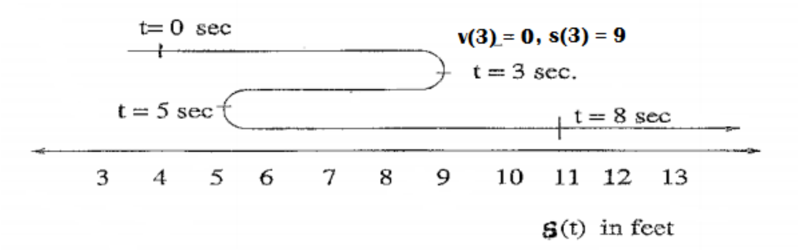
At $t = 3$, the particle is changing directions. The object is moving neither left nor right.



Therefore, the velocity is zero at time = 3 seconds.

Straight Motion

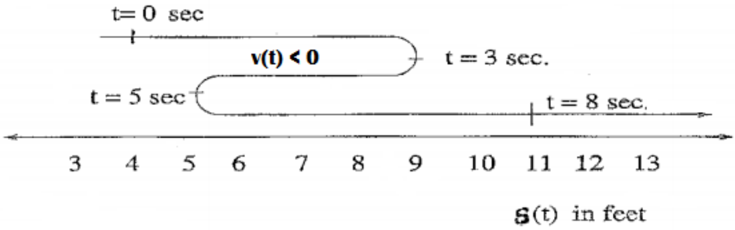
How can we find the position of an object when it is at rest?
We can solve $v(t) = 0$ to find times when the velocity is changing directions.



At $t = 3$ when $v(3) = 0$, calculate $s(3)$ to find the position at rest.

Straight Motion

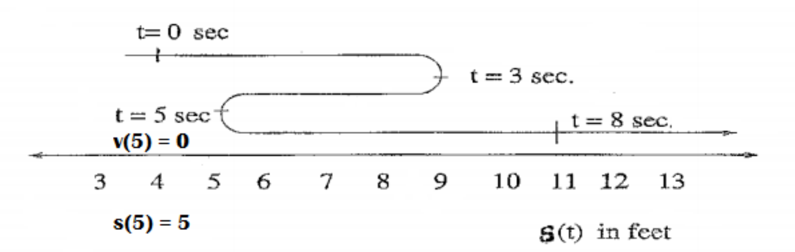
Now, the object has changed direction and is moving to the left.



Therefore, the velocity is negative from $t = 3$ to $t = 5$.

Straight Motion

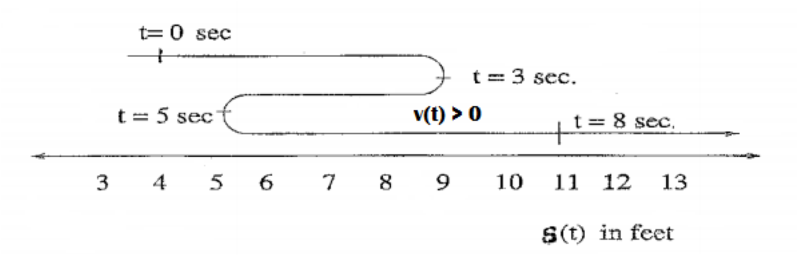
At $t = 5$, again the particle is changing directions. The object is moving neither left nor right.



Therefore, $v(5) = 0$ and $s(5) = 5$.

Straight Motion

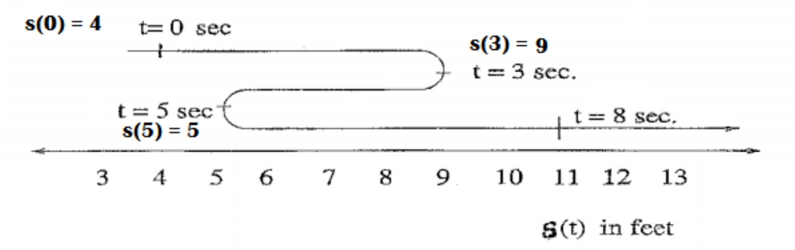
Finally, the object has changed direction and is again moving to the right.



Therefore, the velocity is positive from $t = 5$ to $t = 13$.

Straight Motion

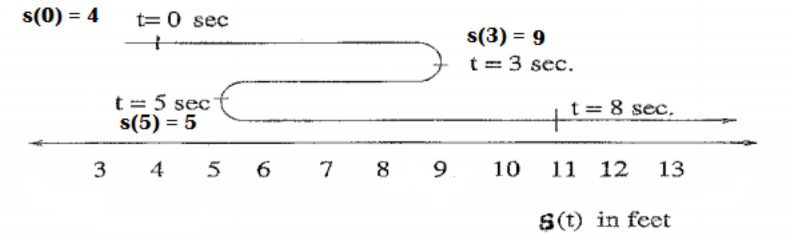
How can we determine displacement over the first 5 seconds?
Displacement is the total distance travelled, not relative distance.



We need to account for distance travelled both forwards ($\rightarrow$) and backwards ($\leftarrow$).

Straight Motion

We know that a change in direction happens at $t = 3$. This means we will have two distance intervals to calculate distance travelled in the interval $t = 0$ to $t = 5$.



$$\text{Distance} = |s(0) - s(3)| + |s(3) - s(5)|$$

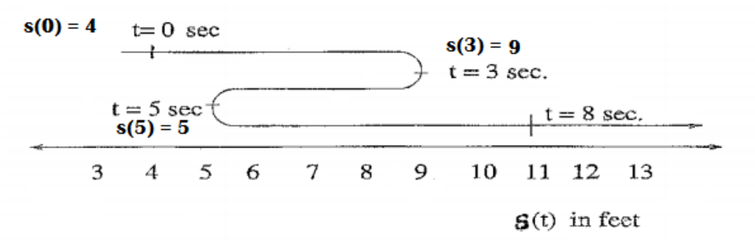
Distance Travelled

The particle has travelled a total of 9 feet in the first 5 seconds.

$$\begin{aligned}
\text{Distance} &= |s(0) - s(3)| + |s(3) - s(5)| \\
&= |4 - 9| + |9 - 5| \\
&= |-5| + |4| \\
&= 9 \text{ feet}
\end{aligned}$$

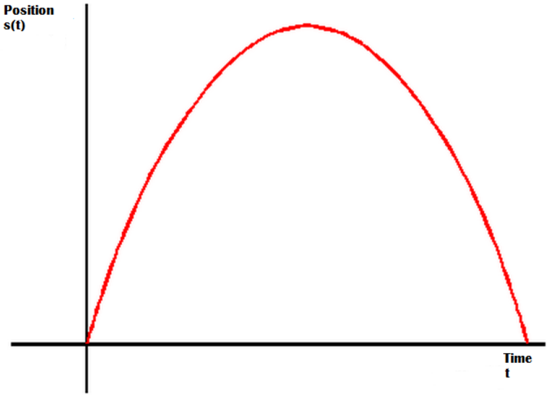
Displacement

What is displacement? Displacement is change in position.

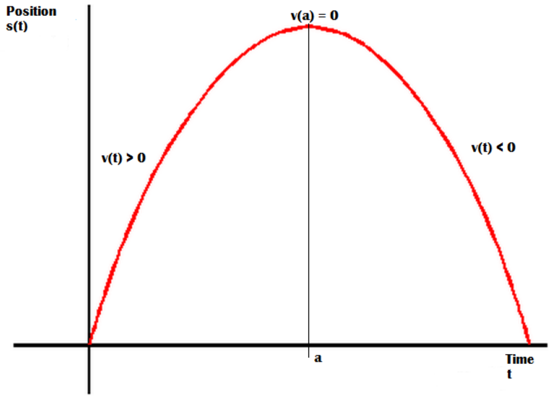


$$\begin{aligned}\text{Displacement} &= s(5) - s(0)\\ &= 5 - 4\\ &= 1 \text{ feet}\end{aligned}$$

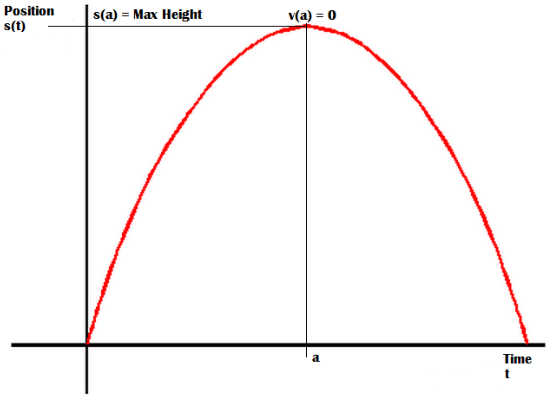
An object with projectile motion has both a vertical (displacement) and a horizontal (time) component.



In a projectile motion graph, the velocity is zero when the object changes directions.



When $v(a) = 0$, the object will reach its maximum height.



Ending Notes

Lab 4 is due next Tuesday, 3/3/2015 in class.

- Proper derivative notation is required throughout the lab.
- Work must be shown to receive credit.
- Tutoring Monday night 5pm-8pm in Jones 112.

These slides are available at www.meganrosebryant.com.