Spring 2015, Math 111
Lab 7: Graphing and the Derivative

Megan Bryant

William and Mary

March 24, 2015

Learning Objectives

Today, we will be looking at applications of linear approximation.

Learning Objectives:

- Review what the first and second derivatives tell us about the graph of a function
- Graphing the derivatives of a function, using derivative information
- Understanding critical points (CPs) and inflection points (IPs)

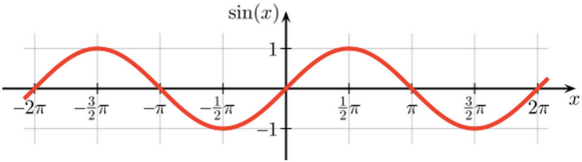
Historical Outline

In the 17th century, Descartes and Fermat revolutionized mathematics by combining algebra and geometry. Their work forms the basis for the graphs that we use in calculus today.

Understanding Graphs

Graphs offer a unique glimpse into the dynamics of a function from the simple line o the oscillatory sine wave.

Technology makes it easy to graph a function. Understanding the function making the connections between the algebra and the geometry is more difficult, however.

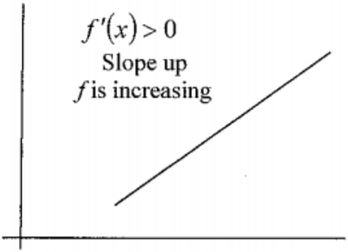


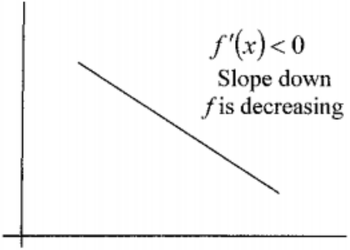
Slope

The first derivative of a function, $f'(x)$, describes the 'slope' of the corresponding graph.

1. If $f'(x) > 0$ for all x in some interval I, then $f(x)$ is increasing up on I.
2. If $f'(x) < 0$ for all x in some interval I, then $f(x)$ is decreasing down on I.

Slope



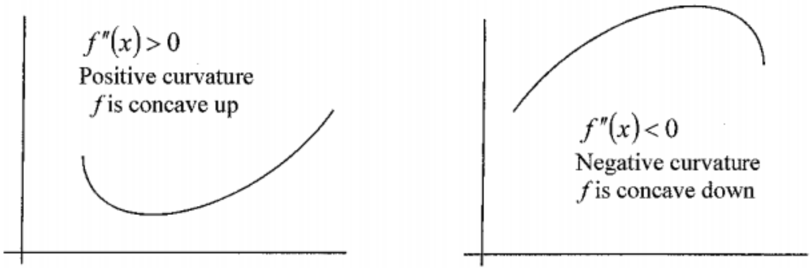


Concavity

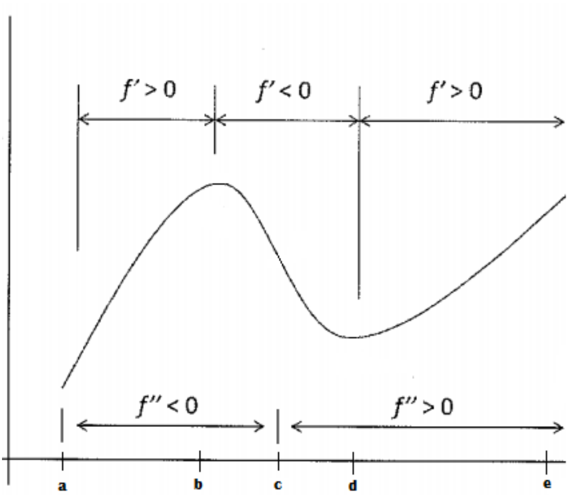
The second derivative of a function, $f'(x)$, describes the 'concavity' of the corresponding graph.

1. If $f''(x) > 0$ for all x in some interval I, then $f(x)$ is concave up on I.
2. If $f''(x) < 0$ for all x in some interval I, then $f(x)$ is concave down on I.

Concavity


f″(x) > 0
Positive curvature
f is concave up
f″(x) < 0
Negative curvature
f is concave down

Slope and Concavity

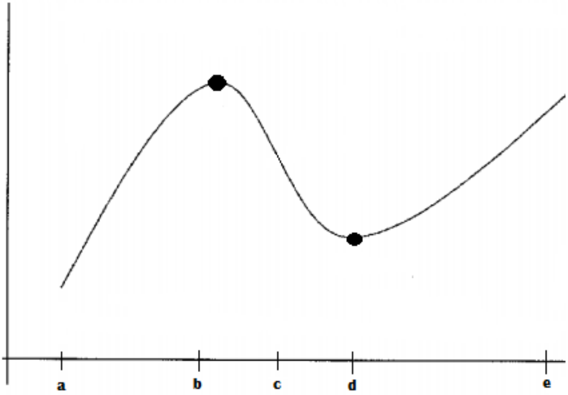


Critical Points

A point $x = c$ is a critical point of the function $f(x)$ if $f(c)$ exists and if either of the following are true:

1. $f'(c) = 0$
   OR
2. $f'(c)$ doesn't exist

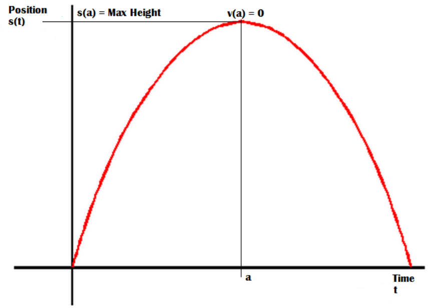
Critical Points



$f'(b) = 0$ and $f'(d) = 0$. Thus, y and z are critical points.

Critical Points

Recall from Lab 3 the problem involving the tossing of the ball in to the air.



At the peak of the ball's parabolic path, the function has a critical point when $v(a) = 0$.

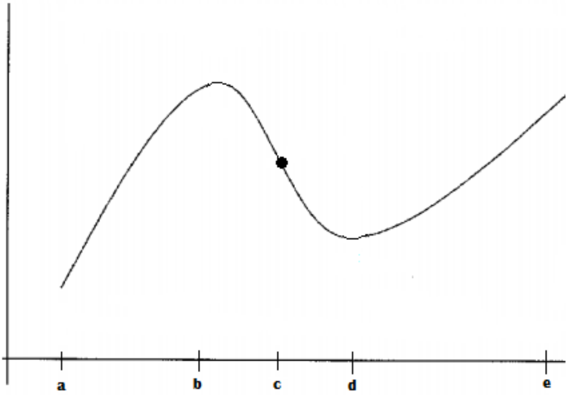
Inflection Points

A point $x = c$ is called an inflection point of the function $f(x)$ if $f(x)$ is continuous at $f(c)$ and if the concavity of the graph changes at that point.

This happens if the second derivative, $f''(x)$ changes from positive to negative or from negative to positive at that point.

Inflection points can be detected by setting the second derivative equal to zero ($f''(x) = 0$) and solving.

Inflection Points



$f''(c) = 0$ and the concavity of the graph changes from negative to positive. Therefore, c is an inflection point.

Example

Find the critical points, inflection points, intervals of increase/decrease, and intervals of concavity of the function $f(x) = 3x^3 - 5x + 3$.

First, find the first derivative of the function.

$$f'(x) = 9x^2 - 5$$

Example

In order to find the critical points, set the first derivative ($f'(x) = 9x^2 - 5$) equal to zero and solve. Use the quadratic formula.

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{0 \pm \sqrt{(0)^2 - 4 * 9 * (-5)}}{2 * 9} \\
&= \pm \frac{\sqrt{180}}{18}
\end{aligned}
$$

Thus, there are two critical points: $\pm \frac{\sqrt{180}}{18}$.

Example

Now, to determine the inflection points begin by deriving the second derivative.

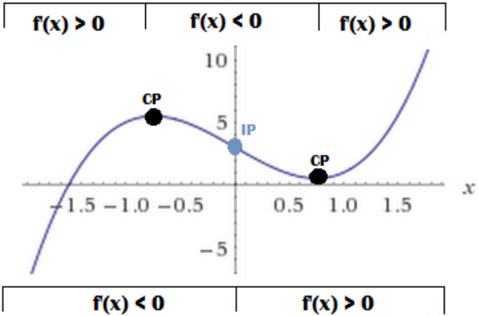
$$f''(x) = 18x$$

Now, set the second derivative equal to zero and solve.

$$18x = 0$$
$$x = 0$$

Thus, there is one inflection point at $x = 0$.

Example



Hints for Lab 7

1. Mark the graph in Exercise 1, using the same notation seen in the example.
2. The problems on page 6 require the quadratic formula.

Ending Notes

Lab 7 is due Tuesday, 3/31/2015 in class.

- Proper derivative notation is required.
- Work must be shown to receive credit.
- Tutoring available Monday night 5pm-8pm in Jones 112.

These slides are available at www.meganrosebryant.com.