

NETWORK OPTIMIZATION

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Executive Summary

Homework 3. Due date: February 12
Collaborators:

3 Dijkstra's Algorithm

Implement Dijkstra's algorithm, or work out the following example by hand. The graph has nodes labeled 1 to 6, and arcs (i, j) for every i and j such that $i < j$. The arc lengths are given in the table. Use Dijkstra's algorithm to find the shortest path from node 1 to node 6.

The original, provided node and edge table:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|-----|-----|------|------|------|
| 1 | | 260 | 570 | 1100 | 1860 | 3360 |
| 2 | | | 450 | 860 | 1500 | 2820 |
| 3 | | | | 460 | 900 | 1920 |
| 4 | | | | | 660 | 1560 |
| 5 | | | | | | 1020 |
| 6 | | | | | | |

The following table illustrates the application of Dijkstra's algorithm iteratively. At each step, the distance to each node from every marked node is calculated and the 'best' (least) distance is noted in the table. Each node that is marked is underlined in the row of the table at which it becomes marked. The predecessor of this node is denoted by its subscript.

Table 1: Dijkstra's Algorithm

| S | d(1) | d(2) | d(3) | d(4) | d(5) | d(6) |
|--------------------|----------|-------------------------|-------------------------|--------------------------|--------------------------|--------------------------|
| \emptyset | <u>0</u> | ∞ | ∞ | ∞ | ∞ | ∞ |
| {1} | | <u>260</u> ₁ | 570 ₁ | 1100 ₁ | 1860 ₁ | 3360 ₁ |
| {1, 2} | | | <u>570</u> ₁ | 1100 ₁ | 1760 ₂ | 3080 ₂ |
| {1, 2, 3} | | | | <u>1030</u> ₃ | 1470 ₃ | 2490 ₃ |
| {1, 2, 3, 4} | | | | | <u>1470</u> ₃ | 2490 ₃ |
| {1, 2, 3, 4, 5} | | | | | | <u>2490</u> ₃ |
| {1, 2, 3, 4, 5, 6} | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |

Therefore, the three nodes that are parents in the shortest path are 1, 3, and 6 with a total minimum distance of 5820. The actual calculations displayed in the above graph and comparisons for each round were done in Excel. Therefore, they are omitted. However, at each round the distance to the remaining unmarked nodes was calculated along the path of each marked node.