CSCI 520
Homework 7

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1. *Identify any typographical errors or suggestions in the first 22 chapters of the R book. Here is a sample format of how to identify the errors. (The header is not included in positive line counts; negative line counts are taken from the bottom of the page.*

- Page 103, line 15. by by → by
- Page 114, line 3. as → as a
- Page 122, line -14. given → give
- Page 133, line 11. User → Use

2. *Add the following abbreviations to your .vimrc file*

```
ab x1n $x_1, x_2, \ldots, x_n$
ab X1n $X_1, X_2, \ldots, X_n$
ab wm The College of William \& Mary
```

Add two more abbreviations of this nature to your .vimrc file that might be helpful in speeding up your LaTeXinput. List the two extra commands you choose for the solution to this problem.

Done.

3. *Write two R commands that calculate* $\sum_{I=1}^{15}\left(\frac{2_i}{i!} - \frac{\cos(3i)}{i^4}\right)$.

R Commands:

```
sum(2^(1:15)/factorial(1:15))-sum(cos(3*1:15)/(1:15)^4)
sum(2^(1:15)/factorial(1:15)-cos(3*1:15)/(1:15)^4)
```

4. *Write two R commands that calculate* $\prod_{x=4}^{12}\left|\frac{x(x-1)(x-2)}{(x-3)!} + \frac{\arctan(x)}{x^2}\right|$.

R Commands:

```
prod((4:12*(4:12-1)*(4:12-2))/(factorial(4:12-3))+atan(4:12)/(4:12)^2)
prod(((4:12)^3-3*(4:12)^2+2*(4:12))/(factorial(4:12-3))+atan(4:12)/(4:12)^2)
```

5. *Let* $x_1, x_2, \ldots, x_n$ *denote the elements of the vector* x. *Write an R function named L2 with a single vector argument* x *that calculates the* L_2 *norm* $\sqrt{x_1^2 + x_1^2 + \ldots + x_n^2}$.
Test your function with the R commands

```
> L2(c(3, 4))
> L2(c(1, 1, 1))
```

R Function:

```
> L2 <- function(x){sqrt(sum(x^2))}
```

Test Values:

```
> L2(c(3,4))
[1] 5
> sqrt(3^2+4^2)
[1] 5
> L2(c(1,1,1))
[1] 1.732051
> sqrt(3)
[1] 1.732051
```

6. *Let $x_1, x_2, \ldots, x_n$ denote the elements of the vector x. Write an R function named Lp with a vector argument x and an integer argument p that calculates the p-norm*

$$\left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}}.$$

Test your function with the R commands

```
> Lp(c(3, 4), 2)
> Lp(c(1, 1, 1), 3)
```

R Commands:

```
> Lp <- function(x,p){(sum(abs(x)^p))^(1/p)}
```

Test Values:

```
> Lp(c(3,4),2)
[1] 5
> Lp(c(1,1,1),3)
[1] 1.44225
```

7. *Write an R command to calculate*

$$\frac{3}{4} + \left(\frac{3}{4} \cdot \frac{5}{6}\right) + \left(\frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8}\right) + \cdots + \left(\frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8} \cdot \cdots \cdot \frac{49}{50}\right).$$

R Command:

```
sum(cumprod((3:49)/(4:50)))
```

8. *Let $x_1, x_2, \cdots, x_n$ be the $n > 2$ elements in vector x. Write an R function named movave with a single argument n that returns a vector of length $n - 1$ whose elements are the moving averages*

$$\frac{x_1 + x_2}{2}, \frac{x_2 + x_3}{2}, \frac{x_3 + x_4}{2}, \ldots, \frac{x_{n-1} + x_n}{2}.$$

Test your function for some sample vectors.

R Function:

```
movave <- function (x)
{(x[1:(length(x)-1)]+x[2:length(x)])/2
}
```

Test Values:

```
> movave(x)
[1]  3  5  7  9 11 13
> x=seq(2,15,by = 2)
> x
[1]  2  4  6  8 10 12 14
> movave(1:5)
[1] 1.5 2.5 3.5 4.5
> movave(2:4)
[1] 2.5 3.5
```

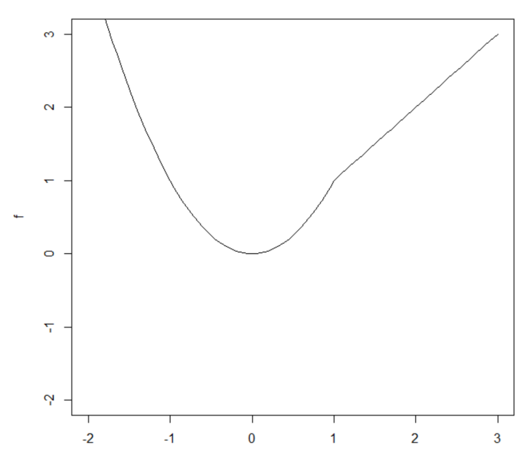
9. *Type help(ifelse) to learn about the ifelse function. Use this function to plot the piecewise function*

$$f(x) = \begin{cases} x^2 & x < 1 \\ x & x \geq 1 \end{cases}$$

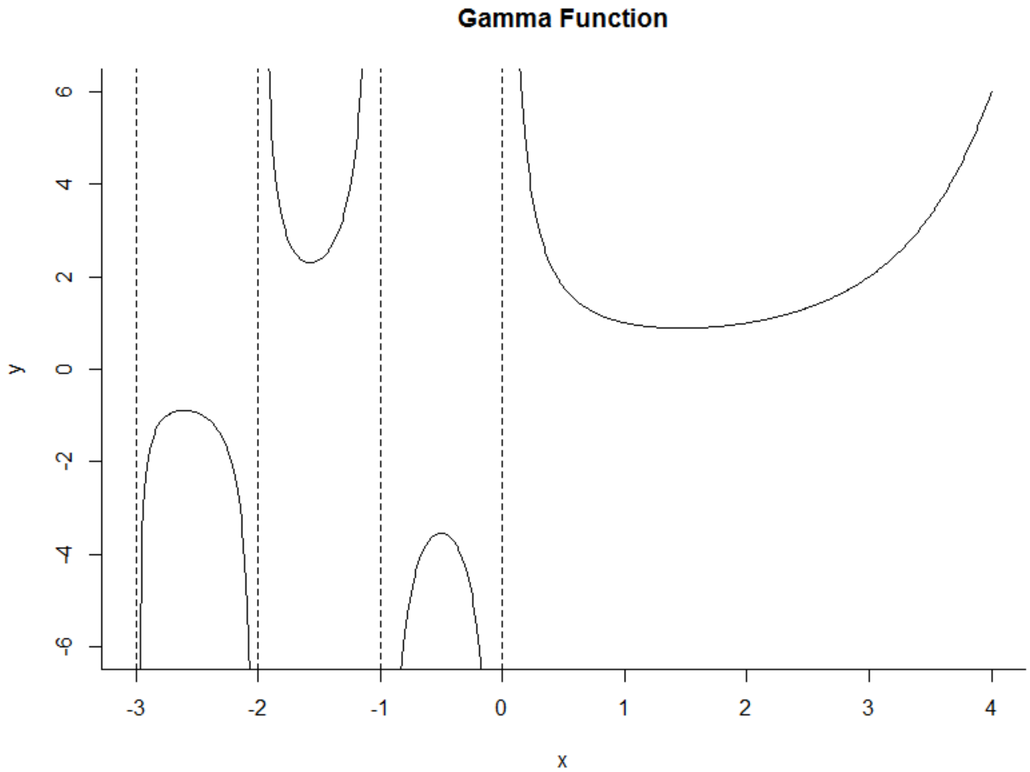
on $-2 < x < 3$ *using a single call to the plot function.*
R Commands:

```
>f <- function (x)
{ifelse(x < 1, x^2, ifelse(x>=1, x, NA))
}
>plot(f, xlim=c(-2,3),ylim=c(-2,3))
```

Graphics Output:



10. *Plot the gamma function $\Gamma(x)$ using R function Gamma for $-3 < x < 4$. Note that the gamma function is undefined for nonpositive integers. Use the limits $-6 \le \Gamma(x) \le 6$ for the vertical axis. Use dashed lines to indicate asymptotes.*



11. *The built-in R function mean calculates the sample mean $\bar{x}$. Write R functions named hmean, gmean, and qmean that calculate the sample harmonic mean, geometric mean, and quadratic mean defined by*

$$h = \left(\frac{1}{n} \sum_{i=1}^{n} \frac{1}{x_i} \right)^{-1} \quad g = \left(\prod_{i=1}^{n} x_i \right)^{\frac{1}{n}} \quad q = \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2}$$

where $x_1, x_2, \ldots, x_n$ are the data values. Test your functions with a vector of data values of your choice and verify the inequality

$$\min\{x_1, x_2, \ldots, x_n\} \le h \le g \le \bar{x} \le q \le \max\{x_1, x_2, \ldots, x_n\}.$$

Test Vector is 1:10.

Harmonic Mean

$$h = \left(\frac{1}{n} \sum_{i=1}^{n} \frac{1}{x_i} \right)^{-1}$$

R Code:

```
Dave: hmean <- function(x){n = length(x)
And?  (1/n*sum(1/x[1:n]))^(-1)}
```

Test:

```
Dave: t = 1:10
Dave: hmean(t)
[1] 3.414172
```

Geometric Mean

$$g = \left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}}$$

R Code:

```
Dave: gmean <- function(x){n = length(x)
And?      (prod(x[1:n]))^(1/n)}
```

Test:

```
Dave: gmean(t)
[1] 4.528729
```

Quadratic Mean

$$q = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}$$

R Code:

```
Dave: qmean <- function(x){n = length(x)
And?  (1/n * sum(x[1:n]^2))^(1/2)}
```

Test:

```
Dave: qmean(t)
[1] 6.204837
```

Verification

$\min\{x_1, x_2, \ldots, x_n\} = 1 \leq h = 3.414172 \leq g = .528729 \leq \bar{x} \leq q6.204837 \leq \max\{x_1, x_2, \ldots, x_n = 10\}$.

We see that the inequality holds true with our calculated test values.