Advanced Calculus II
Unit 8.7: 8.7.1a, 8.7.1c, 8.7.2a, 8.7.3a, 8.7.7, 8.7.8

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8.7.1a

Find the radius of convergence of the following power series: $\sum_{k=1}^{\infty} \frac{3^k}{k^3} x^k$.

We will use the ratio test.

$\frac{1}{R} = \lim_{k\to\infty} \frac{|\frac{3^{k+1}}{(k+1)^3}|}{|\frac{3^k}{k^3}|} = \lim_{k\to\infty} \frac{3^k(3)k^3}{3^k(k+1)^3} = \lim_{k\to\infty} \frac{3k^3}{(k+1)^3} = 3$

Therefore the radius of convergence, $R = \frac{1}{3}$.

8.7.1c

Find the radius of convergence of the following power series: $\sum_{k=1}^{\infty} (1 - \frac{1}{k})^k x^k$.

We will use the root test.

$\frac{1}{R} = \limsup_{n\to\infty} \sqrt[n]{|(1-\frac{1}{n})^n|} = \limsup_{n\to\infty} |(1-\frac{1}{n})| = 1$

Therefore, the radius of convergence, $R = 1$.

8.7.2a

For the following series, determine all values of x for which the series converges: $\sum_{k=0}^{\infty} \frac{k}{x^k}, (x \neq 0)$.

8.7.3a

Use the power series expansion of $\frac{1}{(1-x)}$ *and its derivatives, find* $\sum\limits_{k=1}^{\infty} kx^k, |x| < 1.$

8.7.7

Suppose $f(x) = \sum\limits_{k=0}^{\infty} a_k(x-c)^k$ *has a radius of convergence* $R > 0$*.*

For $|x - c| < R$*, set* $F(x) = \int\limits_c^x f(t)dt$*. Prove that*

$F(x) = \sum\limits_{k=0}^{\infty} \frac{a_k}{k+1}(x-c)^{k+1}, |x-c| < R.$

8.7.8a

Use the previous exercise and the fact that $\frac{d}{dx}$ *Arctan* $x = \frac{1}{1+x^2}$ *to obtain the Taylor series expansion of Arctan* x *about* $c = 0$*.*

8.7.8b

Use part (a) to obtain a series expansion for π*.*

8.7.8c

How large must n *be chosen so that the* n*th partial sum of the series in part (b) provides an approximation of* π *correct to four decimal places?*