Modern Algebra
Homework 2

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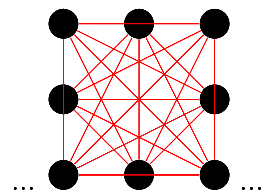
1.14

a.) If only even numbers were allowable,we would still have a group. There is still a predetermined list of actions, every action is reversible, every action is deterministic, and any chain of actions is also an action. This still holds because we are selecing from even whole numbers; it is easily provable that the addition of two even, whole numbers results in an even, whole number. This would not be true if we were instead working with odd, whole numbers because the addition of two odd whole numbers results in an even number, which would violate the 'any chain of actions is also an action' rule.

b.) If we only allowed whole numbers between 0 and 10, we would no longer have a group. While there would still be a predefined list of reversible, and deterministic actions, it would not necessarily be the case that a chain of actions would also be an action. If we started with 1 and, as our first action, added -2, we would have -1, which would be outside the allowable set of whole numbers. Thus, it is no longer a group.

c.)If we changed the parameters to allow all whole numbers, but changed the allowable action to multiplying by a whole number, we would no longer have a group. While there would still be a predetermined list of deterministic actions that can be chained, these actions would be irreversible. This is because if we multiply any whole number by another whole number, the only way to return to the original number by multiplication would be to multiply the resultant number by the original multiple's inverse. To illustrate, $\frac{1}{c} * cx = x$, where c is the whole number multiple and x is the original whole number.

d.)If we allowed only the numbers 1 and -1 and our allowable actions were to multiply be either 1 or -1, we would still have a group. This is because we still have a predermined list of deterministic, reversable actions that can be chained. The following Cayley diagram illustrates this.



2.2

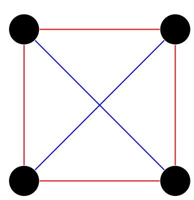
In the light switch puzzle, we have a total of four actions, two of which are generators. The generators are *flip switch 1* and *flip switch 2*. The other two actions are *do nothing* and *flip switch 1 and flip switch 2*.

2.6

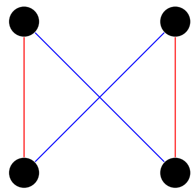
The two groups would have the same Cayley diagram because they are structurally the same.

2.9

a.)

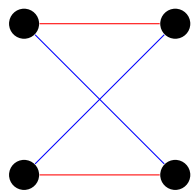


b.)



v and b are sufficient to generate V_4 because every node can be reached by every other node and no node is a dead node.

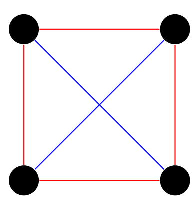
c.)



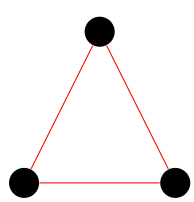
h and b are sufficient to generate V_4 because every node can be reached by every other node and no node is a dead node.

2.11

Another group that would have this Cayley diagram is a mattress with the following actions: *flip horizontally*, *flip vertically*, and *flip diagonally*.



2.12



Since all of the lines are red, we can assume that a single action transfers us between the three states. A group with this Cayley diagram could involve three marbles in a row on a desk with the available action being *pick up the left most marble and put it in the right most position.*

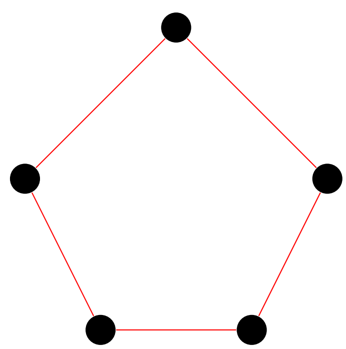
2.17

When using a Cayley diagram as a map for a group, we depend upon the fact that any sequence of consecutive actions is also an action. This is because a Cayley diagram makes no distinction between the order or frequency of an action.

2.19

a.) Based on the cases for $n = 3$ and $n = 4$, the number of generators available for an n-gon would be $2n$.

b.)

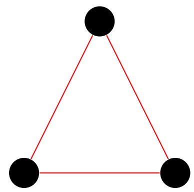


c.i.) Yes, they do.

c.ii.) Yes, the numberof generators for a n-gon is n.

d.) Because an n-gon has n nodes, the smallest number of generators is n. This is because n is the smallest number of "arrows" needed to connect the nodes and create a connected graph.

2.18



The possible action is to move from one node to another in a rotation.

Additional Problem