MTHSC 4420
Advanced Mathematical Programming Homework 3
13-3, 13-6, 13-17, 13-19, 13-21, 13-26, 13-27b, 13-27d, 13-32, 13-34

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13-3

An oil drilling company wishes to locate a supply base somewhere in the jungle area where it is presently exploring for oil. The base will service drilling sites at map coordinates $(0, -30), (50, -10), (70, 20),$ *and* $(30, 50)$ *with helicopter supply runs. The company wishes to choose a location that minimizes the sum of flying distances to the four sites.*

a.)*Formulate an unconstrained NLP to choose an optimum base location.*

Let:
$C = \{1, 2, 3, 4\}$
(x, y) = coordinates of base
(x_i, y_i) = coordinates of drill site i
$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}$

minimize $\sum d_i, i \in C$

b.)*Use class optimization software to compute at least a local optimum starting from coordinates* $(10, 10)$.

AMPL Code

AMPL Mod File

```
param sites = 4;

param A {i in 1..sites};
param B {i in 1..sites};

var x;
var y;
var d{i in 1..sites} = sqrt((x-A[i])^2 + (y-B[i])^2);

minimize distance:sum{i in 1..sites}d[i];
```

AMPL Dat File

```
param A:= 1 0 2 50 3 70 4 30;
param B:= 1 -30 2 -10 3 20 4 50;
```

AMPL Run File

```
reset;
model prob13-3mod.mod;
data prob13-3dat.dat;
option solver conopt;
solve;
display distance;
display x;
display y;
```

AMPL Solution

```
ampl: include run.run;
ampl: CONOPT 3.15C: Locally optimal; objective 149.2688059
9 iterations; evals: nf = 9, ng = 7, nc = 0, nJ = 0, nH = 1, nHv = 6
distance = 149.269

x = 45.7692
y = 2.69231
```

13-6

The following shows a series of measurements of the height (in inches) of a new genetically engineered tomato plant versus the number of weeks after the plant was replanted outdoors.

Week	1	2	4	6	8	10
Height	9	15	22	33	44	52

Researches wish to fit this experience with an S-shaped logistic curve $size = \frac{k}{1+e^{a+b(weeks)}}$ *where* $k, a,$ *and* b *are empirical parameters.*

a.)*Formulate an unconstrained NLP to fit this form to the given data so as to minimize the sum of squared errors.*

Let:
$W = \{1, 2, 4, 6, 8, 10\}$
$H = \{9, 15, 22, 33, 44, 52\}$
$x_i = \frac{k}{1+e^{a+b(W_i)}}$
$y_i = H_i$
$n = 6$

$$\min \sum_{i=1}^{n} (y_i - x_i)^2$$

b.)*Use class optimization software to compute at least a local optimum for your curve fitting model starting with* $k = 50$, $a = 3$, *and* $b = -.03$.

AMPL Code
AMPL Mod File

```
param entries = 6;

param W {i in 1..entries};
param H {i in 1..entries};

var a;
var b;
var k;
var x{i in 1..entries} = k/(1 + (2.718281828)^(a + b*(W[i])));

minimize sum_squared_errors:sum{i in 1..entries}(H[i] - x[i])^2;
```

AMPL Dat File

```
param W:= 1 1 2 2 3 4 4 6 5 8 6 10;
param H:= 1 9 2 15 3 22 4 33 5 44 6 52;
```

AMPL Run File

```
reset;
model 13-6mod.mod;
data 13-6dat.dat;
option solver conopt;
solve;
display sum_squared_errors;
display a;
display b;
display k;
```

AMPL Solution

```
ampl: include run.run;
CONOPT 3.15C: Locally optimal; objective 3.805541881
15 iterations; evals: nf = 17, ng = 13, nc = 0, nJ = 0, nH = 7, nHv = 6
sum_squared_errors = 3.80554

a = 2.01701

b = -0.348056

k = 64.1103
```

13-17

Use golden section Algorithm 13A to find an optimum of the NLP
min $10x + \frac{70}{x}$
s.t. $1 \leq x \leq 10$
to within an error of $+/-1$.

$t = 0$

$x^{(1)} = x^{(hi)} - \alpha(x^{(hi)} - x^{(lo)}) = 10 - \alpha(10 - 1) = \frac{11}{2} - \frac{9\sqrt{5}}{2} \approx -4.56$

$x^{(2)} = x^{(lo)} + \alpha(x^{(hi)} - x^{(lo)}) = 1 + \alpha(10 - 1) = \frac{11}{2} + \frac{9\sqrt{5}}{2} \approx 15.56$

$f(x^{(lo)}) = 10x^{(lo)} + \frac{70}{x^{(lo)}} = 10 * (1) + \frac{70}{1} = 80$

$f(x^{(hi)}) = 10x^{(hi)} + \frac{70}{x^{(hi)}} = 10 * (10) + \frac{70}{10} \approx 107$

$f(x^{(1)}) = 10x^{(1)} + \frac{70}{x^{(1)}} \approx 10 * (-4.56) + \frac{70}{-4.56} \approx -60.95$

$f(x^{(2)}) = 10x^{(2)} + \frac{70}{x^{(2)}} \approx 10 * (15.56) + \frac{70}{15.56} \approx 160.10$

Therefore, $t = t + 1 = 1$.

Therefore $x^{(2)} \rightarrow x^{(hi)}$, $x^{(1)} \rightarrow x^{(2)}$, and $(x^{(hi)} - \alpha(x^{(hi)} - x^{(lo)}) \rightarrow x^{(1)}$.

So we have:
$x^{(hi)} = \frac{11}{2} + \frac{9\sqrt{5}}{2}$
$x^{(2)} = \frac{11}{2} - \frac{9\sqrt{5}}{2}$
$x^{(1)} = x^{(hi)} - \alpha(x^{(hi)} - x^{(lo)} = \frac{11}{2} + \frac{9\sqrt{5}}{2} - \alpha(\frac{11}{2} + \frac{9\sqrt{5}}{2} - 1) = -8$

$(x^{(hi)} - x^{(lo)}) = (\frac{11}{2} + \frac{9\sqrt{5}}{2} - 1) \approx 14.56 \not\leq \epsilon$

$f(x^{(1)}) = 10 * (-8) + \frac{70}{-8} = -88.75$

$f(x^{(2)}) = 10 * (-4.56) + \frac{70}{-4.56} \approx -60.9$

Therefore, $t = t + 1 = 2$.

So we have:
$x^{(hi)} = \frac{11}{2} - \frac{9\sqrt{5}}{2} \approx -4.56$
$x^{(2)} = -8$
$x^{(1)} = x^{(hi)} - \alpha(x^{(hi)} - x^{(lo)} = \frac{11}{2} - \frac{9\sqrt{5}}{2} - \alpha(\frac{11}{2} - \frac{9\sqrt{5}}{2} - 1) \approx 4.44$

$(x^{(hi)} - x^{(lo)}) = (-4.56 - 1) \approx 14.56 \not\leq \epsilon$

$f(x^{(1)}) = 10 * (-8) + \frac{70}{-8} = -88.75$

$f(x^{(2)}) = 10 * (-4.56) + \frac{70}{-4.56} \approx -60.9$

$f(x^{(1)}) \leq f(x^{(2)})$

$x^{(hi)} = -8$
$x^{(1)} = -1.13$
$x^{(2)} = -8 - \alpha(-8 - 1) = 6.56$

$f(x^{(1)} = 10 * -1.13 + \frac{70}{-1.13} = -73.25$

$t = 3$

$x^{(hi)} - x^{(lo)} = -8 - 1 = -9 \nleq 1$

Therefore, we have reached an approximate optimum objective function:
$x^* = \frac{1}{2}(1 + -8) = -3.5$

13-19

Suppose that we were given only the lower limit of 1 *in the NLP of exercise* $13 - 17$*. Apply 3-point pattern algorithm 13B to compute a corresponding upper limits with which golden section search could begin using each of the following initial step size* δ*.*

a.)$\delta = 0.5$

$x^{(lo)} = 1$
$\delta = 0.5$

$f(x^{(lo)}) = 10 * x^{(lo)} + \frac{70}{x^{(lo)}} = 10 + 70 = 80$
$f(x^{(lo)+\delta}) = 10 * (\frac{3}{2}) + \frac{70}{\frac{3}{2}} = 60\frac{2}{3}$

$f(x^{(lo)+\delta}) < f(x^{(lo)})$.

Therefore, $x^{(mid)} = x^{(lo)} + \delta = \frac{3}{2}$.

$\delta = 2 * \delta = 1$

$f(x^{(mid)}) = 10 * \frac{3}{2} + \frac{70}{\frac{3}{2}} = 61\frac{2}{3}$
$f(x^{(mid)}) + \delta) = 10 * \frac{5}{2} + \frac{70}{\frac{5}{2}} = 53$

$f(x^{(mid)}) > f(x^{(mid)}) + \delta)$.

Therefore, we will update:
$x^{(lo)} = \frac{3}{2}$
$x^{(mid)} = \frac{3}{2} + 1 = \frac{5}{2}$
$\delta = 2\delta = 2 * 1 = 2$

$f(x^{(mid)}) = 10 * \frac{5}{2} + \frac{70}{\frac{5}{2}} = 53$
$f(x^{(mid)}) + \delta) = 10 * \frac{9}{2} + \frac{70}{\frac{9}{2}} = 60.5$

Now we have $f(x^{(mid)})$ superior to $f(x^{(mid)}) + \delta)$.

Therefore, we will update $x^{(hi)} = x^{(mid)} + \delta = \frac{9}{2}$.

Our 3-point pattern is $[1, \frac{5}{2}, \frac{9}{2}]$.

b.)$\delta = 16$

$x^{(lo)} = 1$
$\delta = 16$

$f(x^{(lo)}) = 10 * x^{(lo)} + \frac{70}{x^{(lo)}} = 10 + 70 = 80$
$f(x^{(lo)} + \delta) = 10 * (17) + \frac{70}{17} = 174.12$

$f(x^{(lo)} + \delta) > f(x^{(lo)})$.

Therefore $x^{(hi)} = x^{(lo)} + \delta = 17$.

$\delta = \frac{1}{2}\delta = 8$

$f(x^{(lo)}) = 10 * x^{(lo)} + \frac{70}{x^{(lo)}} = 10 + 70 = 80$
$f(x^{(lo)} + \delta) = 10 * (9) + \frac{70}{9} = 97.78$

$f(x^{(lo)} + \delta) > f(x^{(lo)})$.

Therefore $x^{(hi)} = x^{(lo)} + \delta = 9$.

$\delta = \frac{1}{2}\delta = 4$

$f(x^{(lo)}) = 10 * x^{(lo)} + \frac{70}{x^{(lo)}} = 10 + 70 = 80$
$f(x^{(lo)} + \delta) = 10 * (5) + \frac{70}{5} = 64$

$f(x^{(lo)} + \delta) < f(x^{(lo)})$.

Therefore $x^{(mid)} = x^{(lo)} + \delta = 5$.

Our 3-point pattern is $[1, 5, 9]$.

13-21

Use a quadratic fit Algorithm 13C to compute an optimum for the following NLP within an error tolerance of 2. *Start with the 3-point pattern* $\{1, 2, 10\}$*:*
min $10x + \frac{70}{x}$
s.t. $1 \leq x \leq 10$
to within an error of $+/-2$

$x^{(hi)} = 10$

$x^{(mid)} = 2$
$x^{(lo)} = 1$
$f^{(lo)} = f(x^{(lo)}) = 10 * 1 + \frac{70}{1} = 80$
$f^{(mid)} = f(x^{(mid)}) = 10 * 2 + \frac{70}{2} = 55$
$f^{(hi)} = f(x^{(hi)}) = 10 * 10 + \frac{70}{10} = 107$
$s^{(lo} = (x^{(lo)})^2 = 1$
$s^{(mid} = (x^{(mid)})^2 = 4$
$s^{(hi} = (x^{(hi)})^2 = 100$

$(x^{(hi)} - x^{(lo)}) = 10 - 1 = 9 \not\leq 2$

$x^{(qu)} = \frac{1}{2} \frac{f^{(lo)}[s^{(mid)} - s^{(hi)}] + f^{(mid)}[s^{(hi)} - s^{(lo)}] + f^{(hi)}[s^{(lo)} - s^{(mid)}]}{f^{(lo)}[x^{(mid)} - x^{(hi)}] + f^{(mid)}[x^{(hi)} - x^{(lo)}] + f^{(hi)}[x^{(lo)} - x^{(mid)}]} = \frac{1}{2} \frac{80[4-100]+55[100-1]+107[1-4]}{80[2-10]+55[10-1]+107[1-2]} =$
$\frac{71}{14}$

13-26

Consider the 2-variable function $f(x_1, x_2) =^{\Delta} 13x_1 - 6x_1x_2 + \frac{8}{x_2}$ *with current point* $\boldsymbol{x} = (2, 1)$*, and direction* $\Delta x = (3, 1)$

a.)*Derive the first order Taylor approximation to* $f(\boldsymbol{x} + \lambda\Delta x)$.

$f_1(\mathbf{x} + \lambda\Delta x) =^{\Delta} f(x) + \lambda\nabla f(x) * \Delta x$

$$\nabla f(x) = \begin{pmatrix} 13 - 6x_2 \\ -6x_1 - \frac{8}{x^2} \end{pmatrix}$$

$$\nabla f((2,1)) = \begin{pmatrix} 13 - 6 \\ -12 - 8 \end{pmatrix} = \begin{pmatrix} 7 \\ -20 \end{pmatrix}$$

$f_1(\mathbf{x} + \lambda\Delta x) =^{\Delta} f((2,1)) + \lambda(7, -20) * (3, 1)$
$= (13 * 2 - 6 * 2 + 8) + \lambda(7, -20) * (3, 1) = 22 + \lambda(7, -20) * (3, 1) = 22 + \lambda$

b.)*Derive the second order Taylor approximation to* $f(\boldsymbol{x} + \lambda\Delta x)$.

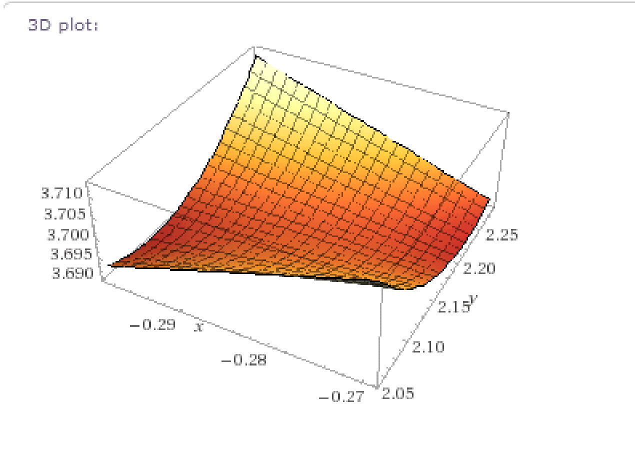
$$H = \begin{pmatrix} \frac{d^2f}{dx_1^2} & \frac{d^2f}{dx_1dx_2} \\ \frac{d^2f}{dx_2x_1} & \frac{d^2f}{dx_2^2} \end{pmatrix} = \begin{pmatrix} 0 & -6 \\ -6 & \frac{16}{x_2^3} \end{pmatrix}$$

$f_2(\mathbf{x} + \lambda\Delta x) =^{\Delta} f((2,1)) + \lambda\nabla f((2,1)) * \Delta x + \frac{\lambda^2}{2}\Delta x\mathbf{H}(x^{(t)})\Delta x =$

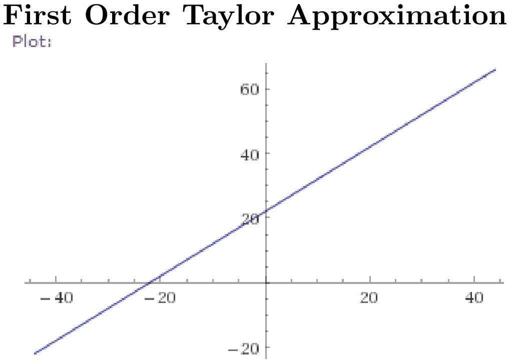
$$22 + \lambda(7, -20) * (3, 1) + \frac{\lambda^2}{2}(3, 1)\begin{pmatrix} 0 & -6 \\ -6 & \frac{16}{x_2^3} \end{pmatrix}(3, 1) = 22 + \lambda + \frac{\lambda^2}{2}(\frac{16}{x_2^3} - 36)$$

c.) *Plot the original function and both Taylor series approximations as function of λ. How accurate do the approximations appear to be? Which is better?*

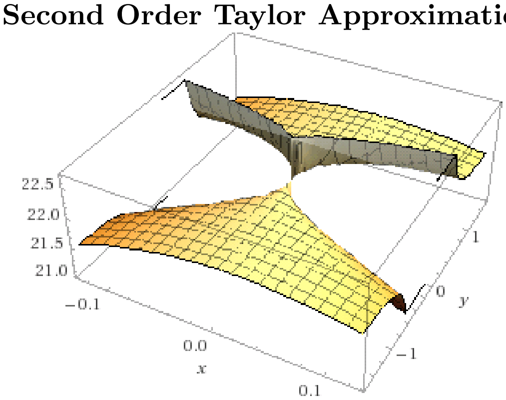
Original Function



First Order Taylor Approximation



Second Order Taylor Approximation



The second order is the better approximation for the function.

13-27b

For the following unconstrained NLP, either verify that the given $\boldsymbol{x}$ *is a stationary point of the bjective function or give a direction* Δx *that improves* x*:* $f(x_1, x_2)\Delta = -(x_1)^2 - 6x_1x_2 - 9(x_2)^2, x = (-3, 1)$.

$$\nabla f(-3,1) = \begin{pmatrix} -2x_1 - 6x_2 \\ -6x_1 - 18x_2 \end{pmatrix} = \begin{pmatrix} -2(-3) - 6 * 1 \\ -6(-3) - 18 * 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Therefore, x is a stationary point.

13-27d

For the following unconstrained NLP, either verify that the given x *is a stationary point of the bjective function or give a direction* Δx *that improves* x*:* $f(x_1, x_2)\Delta = 12x_2 - (x_1)^2 + 3x_1x_2 - 3(x_2)^2$, $x = (12, 8)$.

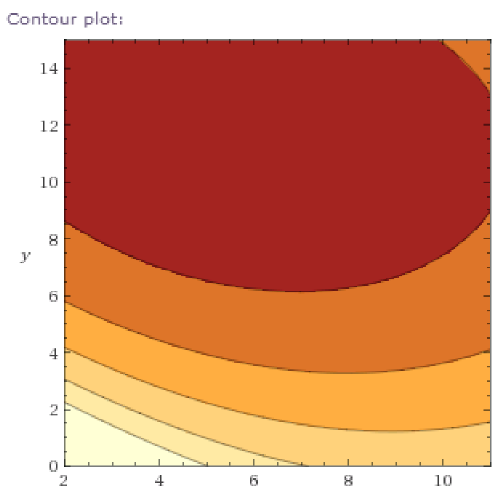
$$\nabla f(-4,10) = \begin{pmatrix} x_2 - 10 \\ x_1 + 4 \end{pmatrix} = \begin{pmatrix} 10 - 10 \\ -4 + 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Therefore, x is a stationary point.

13.32

Consider the unconstrained NLP
$min \frac{1000}{x_1+x_2} + (x_1-4)^2 + (x_2-10)^2$

a.) *Use graphing software to produce a contour map of the objective function for $x_1 \in [2,11]$, $x_2 \in [0,15]$.*



b.) *Compute the move direction that would be pursued by gradient search Algorithm 13D at $x^{(0)} = (10,1)$.*

$x^{(0)} = (10,1)$.

$$\nabla f(x^{(0)}) = \begin{pmatrix} 2(-\frac{500}{(x_1+x_2)^2} + x_1 - 4) \\ 2(-\frac{500}{(x_1+x_2)^2} + x_2 - 10) \end{pmatrix}$$

$$\nabla f(10,1) = \begin{pmatrix} 2(-\frac{500}{(10+1)^2} + 10 - 4) \\ 2(-\frac{500}{(10+1)^2} + 1 - 10) \end{pmatrix} \approx \begin{pmatrix} 3.74 \\ -26.26 \end{pmatrix}$$

$$||\nabla f(x^{(0)})|| =^{\Delta} \sqrt{\sum_j (\frac{df}{dx_j})^2} = \sqrt{(3.74)^2 + (-26.26)^2} = 26.53$$

Therefore the move direction $\Delta x^{(t+1)} = -(3.74, -26.26) = (-3.74, 26.26)$

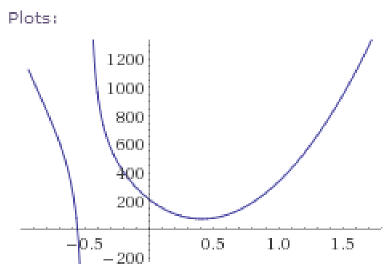
c.) *State the line search problem implied by your direction.*

$\min\ f(x^{(t)} + \lambda \Delta x^{(t+1)})$

d.)*Solve you line search problem graphically and compute the next search point* $x(1)$*.*

$f(x^{(t)} + \lambda \Delta x^{(t+1)}) = f((10, 1) + (-3.74\lambda, 26.26\lambda)) =$
$f(10 - 3.74\lambda, 1 + 26.26\lambda) = (6 - 3.74\lambda)^2 + (26.26\lambda - 9)^2 + \frac{1000}{22.52\lambda+11}$

$\min\ (6 - 3.74\lambda)^2 + (26.26\lambda - 9)^2 + \frac{1000}{22.52\lambda+11} \approx 72.49$



e.)*Do additional interations of Algorithm 13D to compute* $x^{(2)}$ *and* $x^{(3)}$*.*

$x^{(2)} = x^{(1)} + \lambda_{(2)} \Delta x^{(2)} = (10, 1) + \lambda_{(2)}(-3.74, 26.26) =$
$(10 - 3.74\lambda_{(2)}, 1 + 26.6\lambda_{(2)})$.

$$\nabla f((10-3.74\lambda_{(2)}, 1+26.6\lambda_{(2)})) = \begin{pmatrix} 2(-\frac{500}{(10-3.74\lambda_{(2)}+1+26.6\lambda_{(2)})^2} + 10 - 3.74\lambda_{(2)} - 4) \\ 2(-\frac{500}{(10-3.74\lambda_{(2)}+1+26.6\lambda_{(2)})^2} + 1 + 26.6\lambda_{(2)} - 10) \end{pmatrix}$$

Therefore the move direction $\Delta x^{(2)} =$
$(-2(-\frac{500}{(10-3.74\lambda_{(2)}+1+26.6\lambda_{(2)})^2} + 10 - 3.74\lambda_{(2)} - 4), -2(-\frac{500}{(10-3.74\lambda_{(2)}+1+26.6\lambda_{(2)})^2} +$
$1 + 26.6\lambda_{(2)} - 10))$

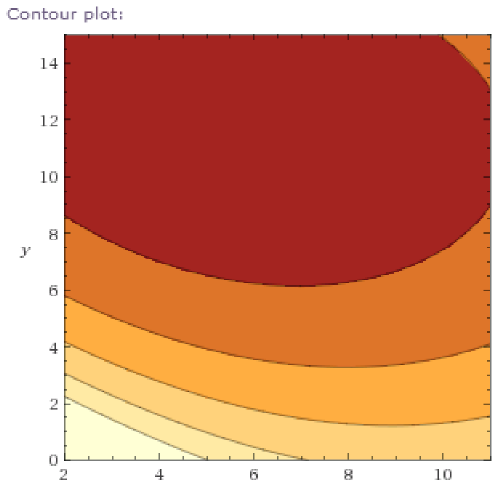
$x^{(3)} = x^{(2)} + \lambda_{(3)} \Delta x^{(2)} = (10 - 3.74\lambda_{(2)} + -2(-\frac{500}{(10-3.74\lambda_{(2)}+1+26.6\lambda_{(2)})^2} + 10 -$
$3.74\lambda_{(2)} - 4), 1 + 26.6\lambda_{(2)} + -2(-\frac{500}{(10-3.74\lambda_{(2)}+1+26.6\lambda_{(2)})^2} + 1 + 26.6\lambda_{(2)} - 10))$.

$\nabla f((10 - 3.74\lambda_{(2)} + -2(-\frac{500}{(10-3.74\lambda_{(2)}+1+26.6\lambda_{(2)})^2} + 10 - 3.74\lambda_{(2)} - 4), 1 +$
$26.6\lambda_{(2)} + -2(-\frac{500}{(10-3.74\lambda_{(2)}+1+26.6\lambda_{(2)})^2} + 1 + 26.6\lambda_{(2)} - 10))) =$

$$\left(\begin{matrix} 2(-\frac{500}{(10-3.74\lambda_{(2)}+-2(-\frac{500}{(10-3.74\lambda_{(2)}+1+26.6\lambda_{(2)})^2}+10-3.74\lambda_{(2)}-4)+1+26.6\lambda_{(2)}+-2(-\frac{500}{(10-3.74\lambda_{(2)}+1+26.6\lambda_{(2)})^2}+1+ \cdots} \\ 2(-\frac{500}{(10-3.74\lambda_{(1)}+1+26.6\lambda_{(2)}+-2(-\frac{500}{(10-3.74\lambda_{(2)}+1+26.6\lambda_{(2)})^2}+1+26.6\lambda_{(2)}-10)))^2} + 1 + 26.6\lambda_{(2)} + -2(-\frac{50 \cdots}{(10-3.74\lambda_{(2)}- \cdots} \end{matrix} \right.$$

Therefore the move direction
$\Delta x^{(3)} = -(2(-\frac{500}{(10-3.74\lambda_{(3)}+1+26.6\lambda_{(3)})^2} + 10 - 3.74\lambda_{(3)} - 4), 2(-\frac{500}{(10-3.74\lambda_{(3)}+1+26.6\lambda_{(3)})^2} +$
$1 + 26.6\lambda_{(3)} - 10))$

f.) *Plot progress of the search on the contour map of part (a).*



13-34

Consider the unconstrained NLP
max $x_1x_2 - 5(x_1-2)^4 - 3(x_2-5)^4$ *starting at* $x^{(0)} = (10,1)$.

a.) *Write the second order Taylor approximation to the ojective function at* $x^{(0)}$ *for unknown* Δx *and* $\lambda = 1$.

$$\nabla f(x_1, x_2) = \begin{pmatrix} y - 20(x_1-2)^3 \\ x_1 - 12(x_2-5)^3 \end{pmatrix}$$

$$\nabla f(10,1) = \begin{pmatrix} 1 - 20(10-2)^3 \\ 10 - 12(1-5)^3 \end{pmatrix} = \begin{pmatrix} -10,239 \\ 778 \end{pmatrix}$$

$$H = \begin{pmatrix} \frac{d^2f}{dx_1^2} & \frac{d^2f}{dx_1dx_2} \\ \frac{d^2f}{dx_2x_1} & \frac{d^2f}{dx_2^2} \end{pmatrix} = \begin{pmatrix} -60(x_1-2)^2 & 1 \\ 1 & -36(x_2-5)^2 \end{pmatrix}$$

$$f_2(\mathbf{x} + \lambda\Delta x) \stackrel{\Delta}{=} f((10,1)) + \nabla f((10,1)) * \Delta x + \tfrac{1}{2}\Delta x \mathbf{H}(x^{(t)})\Delta x =$$
$$-21238 + \begin{pmatrix} -10,239 \\ 778 \end{pmatrix} * \Delta x + \frac{\Delta x}{2} \begin{pmatrix} -60(x_1-2)^2 & 1 \\ 1 & -36(x_2-5)^2 \end{pmatrix} \Delta x$$

b.) *Compute the Newton direction* Δx *at* $x^{(0)}$ *and verify that it is a stationary point of your second-order Talor approximation.*

To find the Newton Direction, we will solve

$f_2(\Delta x) = \nabla f(x^{(t)}) + H(x^{(t)})\Delta x = 0$

$0 = \nabla f(x^{(t)}) + H(x^{(t)})\Delta x = \begin{pmatrix} -10,239 \\ 778 \end{pmatrix} + \begin{pmatrix} -60(x_1-2)^2 & 1 \\ 1 & -36(x_2-5)^2 \end{pmatrix} \Delta x =$

$\begin{pmatrix} -10,239 \\ 778 \end{pmatrix} + \begin{pmatrix} x_2 - 60(x_1-2)^2 x_1 \\ x_1 - 36(x_2-5)^2 x_2 \end{pmatrix} = \begin{pmatrix} -60x_1(x_1-2)^2 + x_2 - 10239 \\ -36x_2(x_2-5)^2 + x_1 + 778 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -60x_1(x_1-2)^2 + x_2 - 10239 \\ -36x_2(x_2-5)^2 + x_1 + 778 \end{pmatrix}$

Which implies that the Newton Direction is $\Delta x = (-\frac{334}{3}, \frac{5}{3})$

c.)*Beginning with your Newton direction, complete 2 iterations of Newton's method Algorithm 13E.*

$x^{(0)} = (10, 1)$
$t = 0$ $\Delta x = (-\frac{334}{3}, \frac{5}{3})$

$\nabla f(10, 1) = \begin{pmatrix} y - 20(x_1-2)^3 \\ x_1 - 12(x_2-5)^3 \end{pmatrix} = \begin{pmatrix} 1 - 20(10-2)^3 \\ 10 - 12(1-5)^3 \end{pmatrix} = \begin{pmatrix} -10,239 \\ 778 \end{pmatrix}$

$H = \begin{pmatrix} \frac{d^2 f}{dx_1^2} & \frac{d^2 f}{dx_1 dx_2} \\ \frac{d^2 f}{dx_2 x_1} & \frac{d^2 f}{dx_2^2} \end{pmatrix} = \begin{pmatrix} -60(x_1-2)^2 & 1 \\ 1 & -36(x_2-5)^2 \end{pmatrix}$

$||\nabla f(x^{(0)})|| =^{\Delta} \sqrt{\sum_j (\frac{df}{dx_j})^2} = \sqrt{(-10239)^2 + 778)^2} = 104837899$

$x^{(1)} = (10, 1) + (\frac{-334}{3}, \frac{5}{3}) = (-101.33, -.66)$

$x^{(1)} = (-101.33, -.66)$
$t = 1$
$\Delta x = (-\frac{334}{3}, \frac{5}{3})$

$\nabla f(-101.33, -.66) = \begin{pmatrix} x_2 - 20(x_1-2)^3 \\ x_1 - 12(x_2-5)^3 \end{pmatrix} = \begin{pmatrix} -.66 - 20(-101.33-2)^3 \\ -101.33 - 12(-.66-5)^3 \end{pmatrix} =$
$\begin{pmatrix} 22065271.26 \\ 2074.53 \end{pmatrix}$

$||\nabla f(x^{(0)})|| =^{\Delta} \sqrt{\sum_j (\frac{df}{dx_j})^2} = \sqrt{(22065271.26 + 2074.53)^2} = 22067345.79$

$x^{(2)} = (-101.33, .66) + (\frac{-334}{3}, \frac{5}{3}) = (-212.66, 2.33)$

d.)*Plot progress of your search on a contour map like the one of Exercise*

13-31(a).