Modern Algebra
Homework 4
Section 4.1
Problems 1, 2, 13, 16, 19-26

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September 13, 2013

4.1

a.) True: $LRRRR = RRL, RLRLRL = LR, and R^8 = R^{100}$.

False:$L \neq RR$.

b.) True: $(LNR)^3 = R^3L^3, NN = N, R^4 = N, and LRLR = N$.

False $(LNR)^2 \neq LNR, RL \neq N$.

c.) R^2.

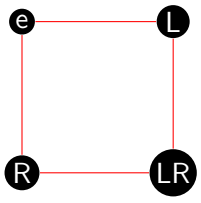
d.) L^2.

e.)$(RL)^2$.

f.)$(LR)^2$

4.2

a.) a.)



b.)

·	e	L	R	LR
e	e	L	R	LR
L	L	e	LR	R
R	R	LR	e	L
LR	LR	R	L	e

4.13

a.) Cannot be a group because the Cayley table is not a latin square.

b.) Cannot be a group because the Cayley table is not a latin square.

c.) Cannot be a group because the Cayley table is not a latin square.

d.) Cannot be a group because there is no identity element.

4.16

a.) Yes, they do meet the requirements of the previous exercises.

b.) Some of the position on the multiplication table require multiple entries.

c.) The actions behave differently depending upon which node they are begining from.

d.)

4.19

a.)

0	1
1	0

Group Z_2

b.)

0	1	2
1	2	0
2	0	1

Group Z_3

c.) 0

Group 1

d.)

0	1	2	3
1	2	3	0
2	3	0	1
3	0	1	2

Group Z_4

e.)

0	1	2	3
1	3	0	2
2	0	3	1
3	2	1	0

Group Z_4

4.20

Klein-4 group:

0	1	2	3
1	0	3	2
2	3	0	1
3	2	1	0

Z_4:

0	1	2	3
1	0	3	2
2	3	1	0
3	2	0	1

4.21

a.) There is one pattern for groups containing three elements.

b.) There is one pattern for groups containing one element.

c.) There are two patterns for groups containing four elements.

4.22

The order matters on the groups S_3 and Quasihedral. Order does not matter for $C_3 \times C_3$, $C_2 \times C_2 \times C_2$ and C_3.

4.23

The smallest group that is noncommutative is S_3.

4.24

There seems to be a type of reflection across the diagonal originating in the bottom left corner in which the colors are changed, but the pattern is the same.

4.25

a.) $Q_4 =< i, j >$.

b.) $A_4 =< a, x >$.

c.) $Q =< -j, -i >$ and $A_4 =< z, d >$.

4.26

a.)
$e^{-1} = e$
$a^{-1} = a^4$
$(a^2)^{-1} = a^3$
$(a^3)^{-1}$
$(a^4)^{-1} = a$

b.)
$1^{-1} = 1$
$i^{-1} = i^3$
$j^{-1} = j^3$
$k^-1 = ji$
$(-1)^{-1} = j^2$
$(-j)^{-1} = j$
$(-i)^{-1} = i$
$(-k)^{-1} = ij$

c.)
$e^{-1} = e$
$x^{-1} = x$
$y^{-1} = axa^2x$
$z^{-1} = axa^2$
$a_1^{-1} = a^2$
$a_2^{-1} = a$
$b_1^{-1} = a^2x$
$b_2^{-1} = xa$
$c_1^{-1} = xa^2$
$c_2^{-1} = ax$
$d_1^{-1} = xa^2x$
$d_2^{-1} = xax$