CSCI 688
Homework 4a

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5.25 *An article in the IEEE Transactions on Electron Devices (Nov. 1986, pp 1754) describes a study on polysilion doping. The experiment shown below is a variation of their study. The response variable is base current.*

Polysilicon	Anneal Temperature(Celsius)		
Doping (ions)	900	950	1000
1×10^{20}	4.60	10.15	11.01
	4.40	10.20	10.58
2×10^{20}	3.20	9.38	10.81
	3.50	10.02	10.60

a.) *Is there evidence with* $\alpha = 0.05$ *indicating that either polysilicon doping level or anneal temperature affects base current?*

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General Linear Model: Base Current versus A, B

Method

Factor coding  (-1, 0, +1)


Factor Information

Factor  Type   Levels  Values
A       Fixed       2  1, 2
B       Fixed       3  1, 2, 3


Analysis of Variance

Source  DF   Seq SS  Contribution   Adj SS   Adj MS  F-Value  P-Value
  A      1    0.980         0.87%    0.980   0.9804    15.26    0.008
  B      2  111.188        98.28%  111.188  55.5940   865.16    0.000
  A*B    2    0.576         0.51%    0.576   0.2879     4.48    0.065
Error    6    0.386         0.34%    0.386   0.0643
Total   11  113.130       100.00%


Model Summary

       S    R-sq  R-sq(adj)   PRESS  R-sq(pred)
0.253492  99.66%     99.38%  1.5422      98.64%
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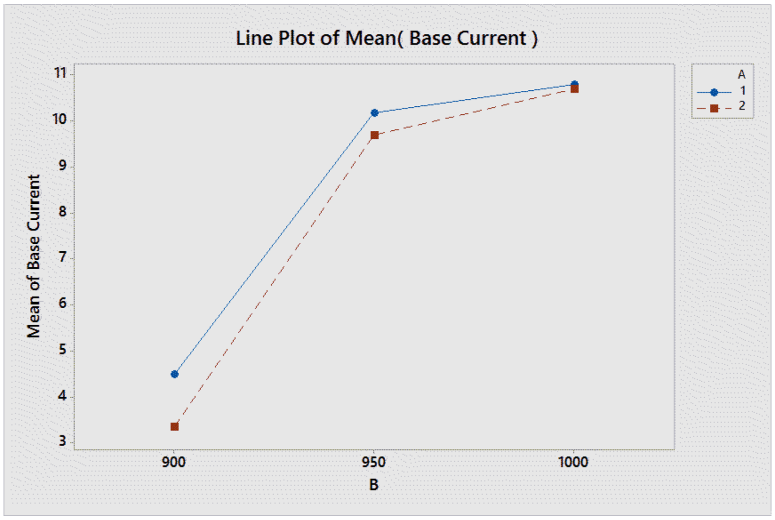
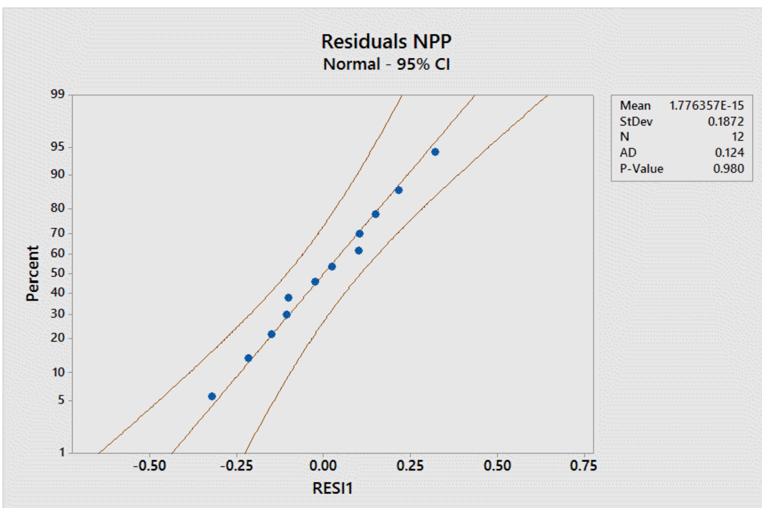
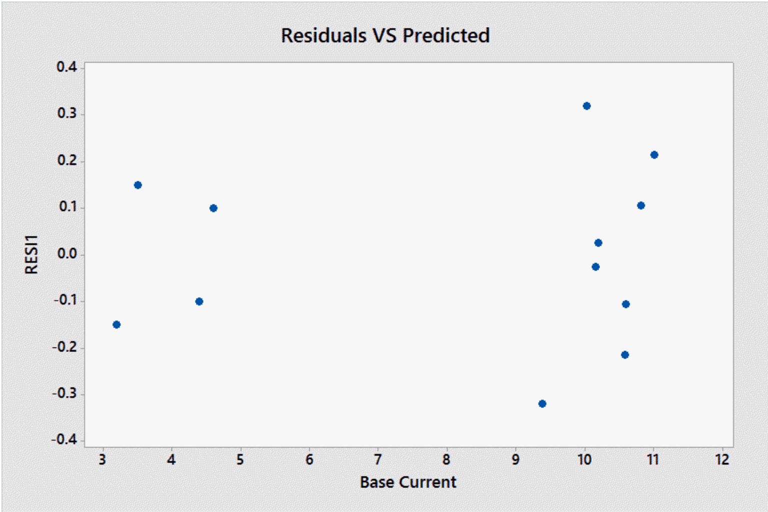
Since the p-values of both the polysilicon doping (factor A) and the anneal temperature (factor b) have p-values of less than $\alpha = 0.05$ with values of $0.008 and 0.000$, respectively, we can conclude that both factors are significant at this level. Their interaction, however is not significant at the 5% level, but would be significant at the 10% level.

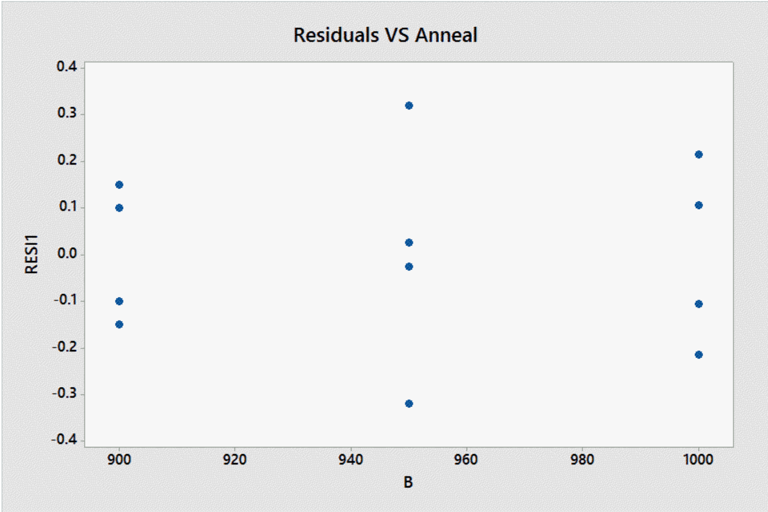
b.) *Prepare graphical displays to assist in interpreting this experiment.*
The following graph illustrates the interaction between the factors and the mean of the base current response variable.

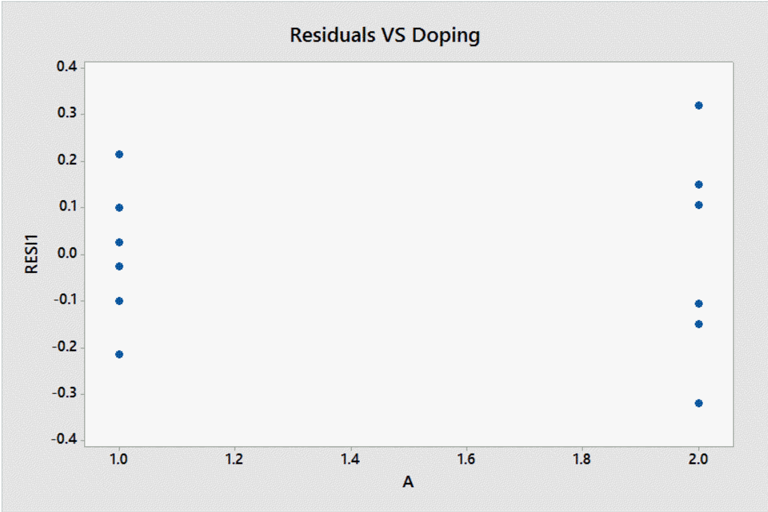


c.) *Analyze the residuals and comment on model adequacy.*









The normal probability plot of the residuals is within acceptable ranges and does not cause us to question the assumption of normality. The residuals vs predicted plot, however, causes us to question the assumption of equal variances due to the odd shape. This odd grouping indicates that there might be some inequality of variances, which would merit further examination.

d.) *Is the model*

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2 + \epsilon$$

supported by the experiment (x_1 = doping level, x_2 = temperature)?. Estimate the parameters in this model and plot the response surface.

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General Linear Model: Base Current versus A, B

Analysis of Variance

Source         DF   Seq SS  Contribution   Adj SS   Adj MS  F-Value  P-Value
  A             1    0.980         0.87%   0.6262   0.6262    10.97    0.013
  B             1   93.161        82.35%  18.9507  18.9507   332.00    0.000
  A*B           1    0.562         0.50%   0.5618   0.5618     9.84    0.016
  B*B           1   18.027        15.93%  18.0267  18.0267   315.81    0.000
Error           7    0.400         0.35%   0.3996   0.0571
  Lack-of-Fit   1    0.014         0.01%   0.0140   0.0140     0.22    0.657
  Pure Error    6    0.386         0.34%   0.3855   0.0643
Total          11  113.130       100.00%


Model Summary

       S    R-sq  R-sq(adj)    PRESS  R-sq(pred)
0.238916  99.65%     99.44%  1.11235      99.02%


Coefficients

Term           Coef   SE Coef          95% CI           T-Value  P-Value       VIF
Constant     -977.5      53.0  (  -1102.8,    -852.3)    -18.46    0.000
A            -10.64      3.21  (   -18.24,     -3.04)     -3.31    0.013    542.50
B             2.028     0.111  (    1.765,     2.292)     18.22    0.000   4342.00
A*B         0.01060   0.00338  (  0.00261,   0.01859)      3.14    0.016    551.50
B*B       -0.001040  0.000059  (-0.001178, -0.000902)    -17.77    0.000   4333.00


Regression Equation

Base Current = -977.5 - 10.64 A + 2.028 B + 0.01060 A*B - 0.001040 B*B
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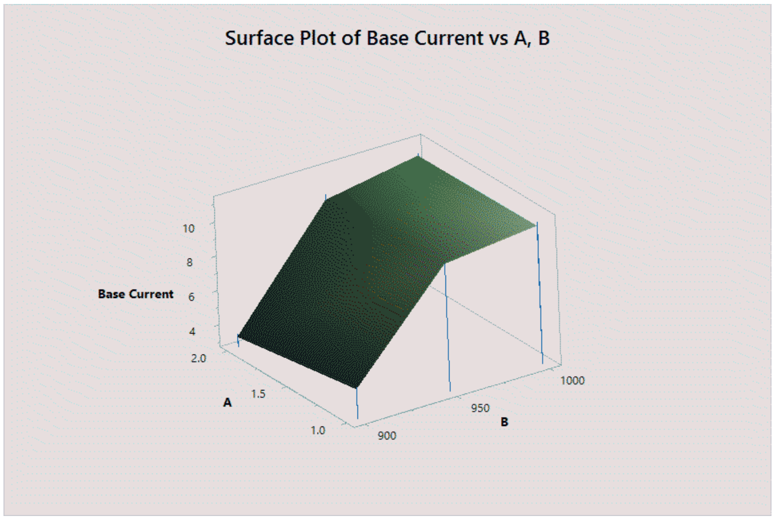
We see that since the p-value is less than our α of 0.05 that all factors are deemed significant in this quadradtic model and therefore all coefficients are significant. These coeffecients are deemed significant and calculated above.



13.1 *A textile mill has a large number of looms. Each loom is supposed to provide the same output of cloth per minute. To investigate this assumption, five looms are chosen at random, and their output is noted at different times. The following data are obtained:*

Loom	Ouput (lb/min)				
1	14.0	14.1	14.2	14.0	14.1
2	13.9	13.8	13.9	14.0	14.0
3	14.1	14.2	14.1	14.0	13.9
4	13.6	13.8	14.0	13.9	13.7
5	13.8	13.6	13.9	13.8	14.0

a.) *Explain why this is a random effects experiment. Are the looms equal in ouput? Use* $\alpha = 0.05$.

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General Linear Model: Output versus Loom

Method

Factor coding  (-1, 0, +1)


Factor Information

Factor  Type    Levels  Values
Loom    Random       5  1, 2, 3, 4, 5


Analysis of Variance

Source  DF  Adj SS   Adj MS  F-Value  P-Value
  Loom   4  0.3416  0.08540     5.77    0.003
Error   20  0.2960  0.01480
Total   24  0.6376

Variance Components, using Adjusted SS
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Source	Variance	% of Total	StDev	% of Total
Loom	0.01412	48.82%	0.118828	69.87%
Error	0.0148	51.18%	0.121655	71.54%
Total	0.02892		0.170059	
Total	24 0.6376			

This is a random effects experiment because the five looms tested were a random sampling of all of the manufacturer's looms. At the 5% significance level, the looms are not equal in output. since the p-value is 0.003.

b.) *Estimate the variabilty between looms.*
The variability between the looms is obtained from the above ANOVA data, but is calculated as follows.

$$\hat{\sigma}_\tau^2 = \frac{MS_{Treatment} - MS_{Error}}{n} = \frac{0.08540 - 0.01480}{5} = 0.01412$$

c.) *Find the experimental error variance.*
We obtained the experimental error variance from the previously included ANOVA data.

$$\hat{\sigma} = MS_{Error} = 0.0148$$

d.) *Find a 95 percent confidence interval for* $\frac{\sigma_\tau^2}{\sigma_\tau^2+\sigma_\tau^2}$
We know that

$$\frac{L}{1+L} \leq \frac{\sigma_\tau^2}{\sigma_\tau^2+\sigma_\tau^2} \leq \frac{U}{1+U}$$

First, we need to find L.

$$L = \frac{1}{n}\left(\frac{MS_{Treatment}}{MS_{Error}}\frac{1}{F_{\alpha/2,\alpha-1,N-\alpha}} - 1\right) = \frac{1}{5}\left(\frac{0.08540}{0.01480} \times \frac{1}{3.51} - 1\right) = 0.1288$$

Next, we must find U.

$$U = \frac{1}{n}\left(\frac{MS_{Treatment}}{MS_{Error}}\frac{1}{F_{1-\alpha/2,\alpha-1,N-\alpha}} - 1\right) = \frac{1}{5}\left(\frac{0.08540}{0.01480} \times \frac{1}{3.51} - 1\right) = 9.6787$$

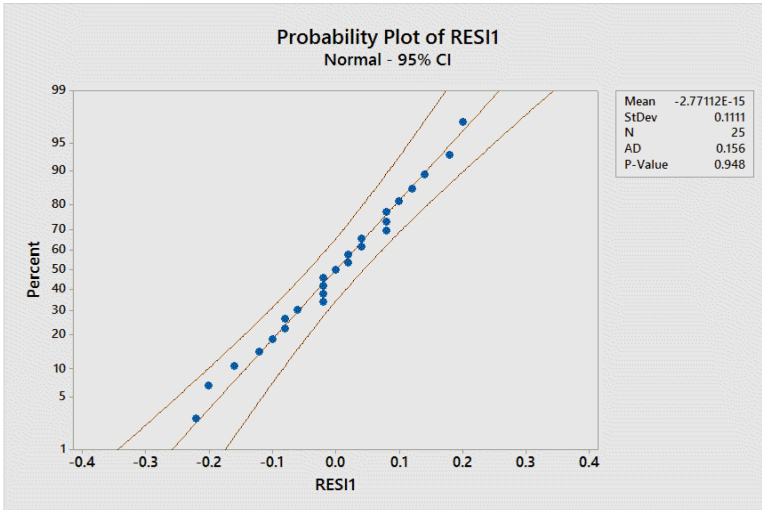
Now, we see that

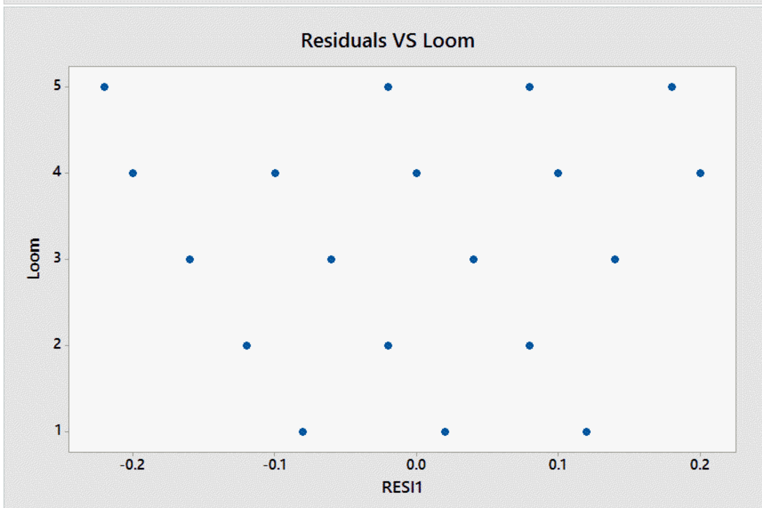
$$\frac{0.1288}{1.1288} \leq \frac{\sigma_\tau^2}{\sigma_\tau^2+\sigma_\tau^2} \leq \frac{9.6787}{10.6787}$$

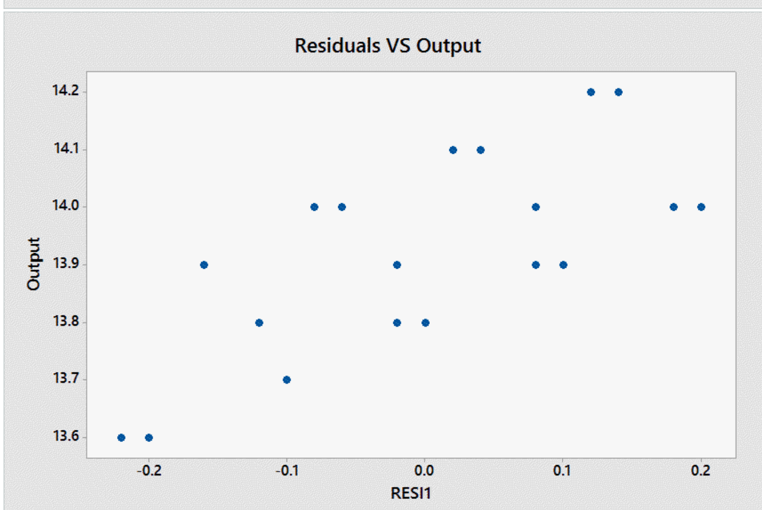
Which reduces to the following 95% confidence interval.

$$0.1141 \leq \frac{\sigma_\tau^2}{\sigma_\tau^2+\sigma_\tau^2} \leq 0.9064$$

e.) *Analyze the residuals from this experiment. Do you think that the analysis of variance assumptions are satisfied.*







Based on the above graphics, we have no reason to question the normality or equal variance assumptions. The normal probability plot has no outliers and passes the fat pencil test. The versus plots show no obvious signs or patterns that give us cause to question our assumptions.

13.8 *Refer to problem 13.1.*

a.) *What is the probability of accepting H_0 if σ_τ^2 is four times the error variance σ^2*
We are looking for the β of the test (the probability of accepting H_0. Therefore, we must first find

$$\lambda = \sqrt{1 + \frac{n\sigma_\tau^2}{\sigma 2}} = \sqrt{1 + \frac{5(4\sigma^2)}{\sigma^2}} = \sqrt{1 + 5 * 4} = \sqrt{21} = 4.58$$

Now, we must determine β from the operating characteristic curve. We know that

$$\nu_1 = a - 1 = 5 - 1 = 4$$

and

$$\nu_2 = N - a = 5 * 5 - 5 = 20$$

We will use an α of 0.05.
Therefore, using the tables in appendix VI, we see that the probability of accepting H_0 given that $\sigma_\tau^2 = 4 * \sigma^2$ is approximately 3.5%, i.e. $\beta = 0.035$.

b.) *If the difference between looms is large enough to increase the standard deviation of an observation by 20 percent, we wish to detect this with a probability of at least 0.80. What sample size should be used?*
We know that to determine the correct sample size, we need to use the following formula:

$$\lambda = \sqrt{1 + n[(1 + 0.01P)^2 - 1]}$$

Where
$\nu_1 = a - 1 = 5 - 1 = 4$
$\nu_2 = N - a = 5 * 5 - 5 = 20$
$\alpha = 0.05$
$P(accepting) \leq 0.2$
Therefore

$$\lambda = \sqrt{1 + n[(1 + 0.01P)^2 - 1]} = \sqrt{1 + n(1.2^2 - 1)} = \sqrt{1 + 0.44n}$$

Now, we must use the operating charecteristic curves in table **VI**. We can see that for our α, we are looking for a $\lambda \in [2.5, 3]$.

n	λ
5	1.77
10	2.32
15	2.76

Therefore, we must select a sample size of $n = 15$ to detect a probability of at least 80%.

13.10 *An article by Hoof and Berman ("Statistical Analysis of Power Module Thermal Test Equipment Performance," IEEE Transactions on Components, Hybrids, and Manufacturing Technology Vol 11, pp 516-520, 1988) describes an experiment conducted to investigate the capability of measurements in thermal impedance ($C/w \times 100$) on a power module for an induction moto starter. There are 10 parts, three operators, and three replicates. The data are shown in Table 13.2.*

a.) *Analyze the data from this experiment, assuming that both parts and operators are random effects.*

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General Linear Model: Response versus Part, Inspector

Method

Factor coding  (-1, 0, +1)


Factor Information

Factor     Type    Levels  Values
Part       Random      10  1, 2, 3, 4, 5, 6, 7, 8, 9, 10
Inspector  Random       3  1, 2, 3


Analysis of Variance

Source            DF   Adj SS   Adj MS  F-Value  P-Value
  Part             9  3935.96  437.328   162.27    0.000
  Inspector        2    39.27   19.633     7.28    0.005
  Part*Inspector  18    48.51    2.695     5.27    0.000
Error             60    30.67    0.511
Total             89  4054.40
```

Based on the above ANOVA results, we see that every factor is significant at the 5% level since all three factors have p-values of less than 0.05.

b.) *Estimate the variance components using the analysis of variance method.*

$$\hat{\sigma}^2 = MS_E = 0.511$$
$$\hat{\sigma}^2_{\tau\beta} = \frac{MS_{AB}-MS_E}{n} = \frac{2.695-0.511}{3} = 0.728$$
$$\hat{\sigma}^2_{\beta} = \frac{MS_B-MS_{AB}}{an} = \frac{437.328-2.695}{3*3} = 48.293$$
$$\hat{\sigma}^2_{\tau} = \frac{MS_A-MS_{AB}}{bn} = \frac{19.633-2.695}{10*3} = 0.5646$$

13.15 *Reanalyze the measurement system experiment in Problem 13.10, assuming that operators are a fixed factor. Estimate the appropriate model components.*

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General Linear Model: Response versus Part, Inspector

Method

Factor coding  (-1, 0, +1)


Factor Information

Factor     Type    Levels  Values
Part       Random      10  1, 2, 3, 4, 5, 6, 7, 8, 9, 10
Inspector  Fixed        3  1, 2, 3
```

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Part	9	3935.96	437.328	162.27	0.000
Inspector	2	39.27	19.633	7.28	0.005
Part*Inspector	18	48.51	2.695	5.27	0.000
Error	60	30.67	0.511		
Total	89	4054.40			

$\hat{\sigma}^2 = MS_E = 0.511$
$\hat{\sigma}^2_{\tau\beta} = \frac{MS_{AB} - MS_E}{n} = \frac{2.695 - 0.511}{3} = 0.728$
$\hat{\sigma}^2_{\beta} = \frac{MS_B - MS_E}{an} = \frac{437.328 - 0.511}{3*3} = 48.535$