Modern Algebra
Homework 5
Chapter 5
Read Problems 5.1,5.11,5.12,5.21,5.22
Do 5.3, 5.10(a,b,c,d), 5.13, 5.14, 5.15(a,e,f),5.30,5.33

Megan Bryant

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5.3

a.)True. Suppose that G is a cyclic group that is generated by the element g. $x, y \in G$. Since G is generated by g, there exists $r, s \in Z$ such that $x = g^r$ and $y = g^s$. Which implies $xy = g^r g^s = g^{r+s} = g^s g^r = yx$. Therefore the group is Abelian.

b.)False, the Klien four-group is Abelian, but not cyclic.

c.)False, D_4 is not Abelian.

d.)False. A group that represents the symmetry of the letter 'A' is cyclic.

e.)True. $C_100 = Z_100$ because there is a cyclic group of every order $n \in N$.

f.)True.S_n can exist by taking all permutations of a given size,$n!$.

g.)False. Every action in the group must be commutative for the group to be Abelian.

h.)False. A group can be Abelian (all actions commute), but not Cyclic.The Klien 4 group is an example.

i.)True. A group that does not have that pattern will be Abelian because it will not matter in what order the actions are taken.

5.10a

The Cayley diagram does represent an Abelian group because VH = HV. It is the direct product group $C_2 \times C_2$. This is called the Klien four group.

5.10b

The Cayley diagram represents an Abelian group because it is the cyclic group C_4. All cyclic groups are Abelian.

5.10c

The Cayley diagram does not represent an Abelian group because $RF \neq FR$. This is a dihedral group D_5.

5.10d

This Cayley diagram is not Abelian as it is D_5.

5.13a

5.13b

5.14

a.) No. Because the order of a dihedral group is $2n$, which would mean that there would have to exist $D_3.5$, which is not possible since $n \in N$.

b.)If A_n has order 2520, then $n = 7$ because $2520 = \frac{7!}{2}$.

c.)If A_n has order m then S_n has order $2m$ because the order of an alternating group of size n is always half the order of the symmetric group of size n.

5.15a

In the group D_{10}, the orbit of element $r^2 = \{e, r^2, r^4, r^6, r^8\}$.

5.15e

The orbit of the element $s = \{a, s, m, j\}$.

5.15f

The orbit of the element $l = \{a, l, e, p\}$.

5.30a

5.30b

5.30c

Yes, the group Z is Abelian because it is cyclic and all cyclic groups are Abelian.

5.30d

No, because it is cyclic, it does not describe a symmetrical group.

5.33a

$< r, f | r^3 = 1, f^2 = 1, frf = r^{-1} >$ gives me the step-by-step process for creating a Cayley diagram for S_3 because it tells me first that there are two arrow colors (r and f). Next, it tells me that performing action r 3 times brings me back to the identity element, so I know that I will have nodes for r and r^2, but not r^3. Next, I see that f^2 returns me to the identity element, which tells me that the f action is an undirected line. That also tells me that the Cayley diagram will have an interior set of 3 nodes. Finally, $frf = r^{-1}$ tells me the direction of the interior r action is counterclockwise. Thus, I have all of the information to build the Cayley diagram.

5.33b

$< r, f | r^4 = 1, f^2 = 1, frf = r^{-1} >$ gives me the step-by-step process for creating a Cayley diagram for D_4 because it tells me first that there are two arrow colors (r and f). Next, it tells me that performing action r 4 times brings me back to the identity element, so I know that I will have nodes for r and r^2, r^3, but not r^4. Next, I see that f^2 returns me to the identity element, which tells me that the f action is an undirected line. That also tells me that the Cayley diagram will have an interior set of 4 nodes. Finally, $frf = r^{-1}$ tells me the direction of the interior r action is counterclockwise. Thus, I have all of the information to build the Cayley diagram. So, this is the presentation for the group D_4.

5.33c

$< a|a^n = 1 >$ is the presentation for the cyclic group C_n.

5.33d

$D_n = < r, f|r^2 = d^n = 1, frf = r^{-1} >$.

5.33e

This is the Quaternion group of order 8 (Q_4).