Modern Algebra
Homework 5
Chapter 5b
Read Problems 5.25-5.27, 5.32, 5.36
Do VGT 5.20, 5.35, 5.37, 5.41(b), 5.42
Additional Questions A, B, C

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5.20

$S_1 = Z_1 = A_2$

$C_2 = S_2 = D_1$

$C_3 = A_3$

$S_3 = D_3$

5.35

a.)The part of the multiplication table that expresses this information is the interior of the table. The part of the Cayley diagram that expresses the same information is seen by starting at node a, doing action b and arriving at node c.

b.)The upper left quarter of the multiplication table for D_5 corresponds to the outer ring of the Cayley diagram because only the elements $\{e, r, r^2, r^3, r^4\}$ are involved and they only interact with eachother. This is essentially the 'non-flip' portion of the diagram.

c.)The lower left quarter of the multiplication table for D_5 corresponds to the inner ring of the Cayley diagram because only the elements

$\{f, fr, fr^2, fr^3, fr^4\}$ are involved and they only interact with eachother. This is essentially the 'flip' portion of the diagram.

d.) The diagonal stripes slant are opposite in the right half of the table as opposed to the left half of the table because the starting element is different. D_5 is non -abelian, so the order of operations matters. $a * b = c \neq b * a = c$.

5.37

a.)The groups of orer six are $S_3 = D_3$ and C_6. C_6 is abelian, but $S_3 = D_3$ is non-abelian.

b.)If there is only one group of any order, then that group must be abelian. This is because the order of the group is prime and the group itself is simple. Therefore, its multiplication table must be symmetric and the group abelian.

c.) The values of n for which there is only one group of order n are prime numbers. For example, $n = 2, 5, 7, 11, \cdots$.

5.41b

1 2 3 4 5 6 7 8 9 10

5.42

In addition to C_1 and all of S_3, the possible subgroups are C_3 and C_2.

A

a.)(1 3 2)(1 2 5 4)(1 5 3) = (4 5)

b.)(1 5)(1 2 4 6)(1 5 4 2 6 3) = (1 4 3)(2)(5 6)

B

$() = (1)(2)(3)(4)$, even

$(1\ \ 2) = (1\ \ 2)(1)(2)$, odd

$(1\ \ 3) = (1\ \ 3)(2)(4)$, odd

$(1\ \ 4) = (1\ \ 4)\ (3)(2)$, odd

$(2\ \ 3) = (2\ \ 3)(1)(4)$, odd

$(2\ \ 4) = (2\ \ 4)(1)(3)$, odd

$(3\ \ 4) = (3\ \ 4)(1)(2)$, odd

$(1\ \ 2)(3\ \ 4) = (1\ \ 2)(3\ \ 4)\ (1\ \ 2)\ (1\ \ 2)$, even

$(1\ \ 3)(2\ \ 4) = (1\ \ 3)(2\ \ 4)\ (1\ \ 3)\ (1\ \ 3)$, even

$(1\ \ 4)(3\ \ 2) = (1\ \ 4)(3\ \ 2)(1\ \ 4)(1\ \ 4)$, even

$(1\ \ 2\ \ 3) = (1\ \ 2\ \ 3)(4)$, even) $= (1\ \ 3)(3\ \ 2)$

$(1\ \ 3\ \ 2) = (1\ \ 3\ \ 2)(4)$, even $= (1\ \ 2)(2\ \ 3)$

$(2\ \ 3\ \ 4) = (2\ \ 3\ \ 4)(1)$,even $= (2\ \ 4)(4\ \ 3)$

$(2\ \ 4\ \ 3) = (2\ \ 4\ \ 3)(1)$, even $= (2\ \ 3)(3\ \ 4)$

$(3\ \ 4\ \ 1) = (3\ \ 4\ \ 1)(2)$, even $= (3\ \ 1)(1\ \ 4)$

$(3\ \ 1\ \ 4) = (3\ \ 1\ \ 4)(2)$, even $= (3\ \ 4)(4\ \ 1)$

$(4\ \ 1\ \ 2) = (4\ \ 1\ \ 2)(3)$, even $= (4\ \ 2)(2\ \ 1)$

$(\ 4\ \ 2\ \ 1) = (\ 4\ \ 2\ \ 1)(3)$, even $= (4\ \ 1)(\ 1\ \ 2)$

$(\ 1\ \ 2\ \ 3\ \ 4) =(1\ \ 4)(1\ \ 3)(1\ \ 2)$, odd

$(1\ \ 2\ \ 4\ \ 3) =(1\ \ 3)(1\ \ 4)(1\ \ 2)$, odd

$(1\ \ 3\ \ 2\ \ 4) = (1\ \ 4)(1\ \ 2)(1\ \ 3)$, odd

$(1\ \ 3\ \ 4\ \ 2) = (1\ \ 2)(\ 1\ \ 4)(1\ \ 3)$, odd

$(1\ \ 4\ \ 2\ \ 3) = (1\ \ 3)(1\ \ 2)(1\ \ 4)$, odd

$(1\ \ 4\ \ 3\ \ 2) = (1\ \ 2)(1\ \ 3)(1\ \ 4)$, odd

C

$A_4 = \{(), (1\,2)(3\,4), (1\,4)(2\,3), (1\,2\,3), (1\,3\,2), (1\,3\,4), (1\,4\,3), (1\,2\,4), (1\,4\,2), (2\,3\,4), (2\,4\,3)\}$ $\blacksquare$