CSCI 688
Homework 6

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7.1 *Consider the experiment described in Problem 6.1. Analyze this experiment assuming that each replicate represents a block of a single production shift.*

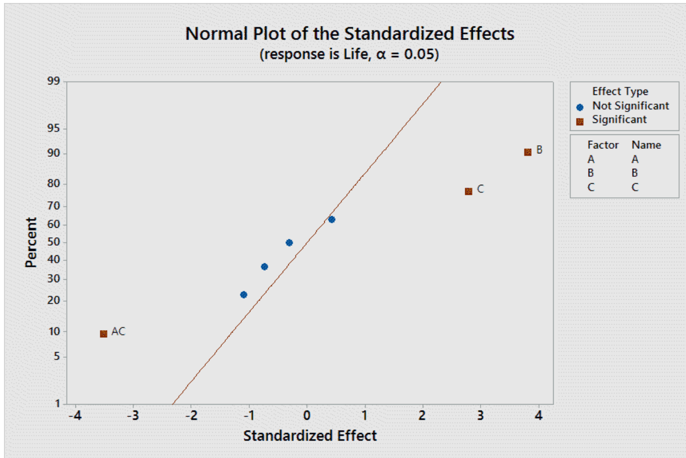
The experiment details and data are copied below for convienance:
6.1 *An engineer is interested in the effects of cutting speed (A), tool geometry (B), and cutting angle (C) on the life (in hours) of a machine tool. Two levels of each factor are chosen, and three replicates of a 2^3 factorial design are run. The results are as follows:*

			Treatment	**Replicate**		
A	**B**	**C**	**Combination**	**I**	**II**	**III**
-	-	-	(1)	22	31	25
+	-	-	a	32	43	29
-	+	-	b	35	34	50
+	+	-	ab	55	47	46
-	-	+	c	44	45	38
+	-	+	ac	40	37	36
-	+	+	bc	60	50	54
+	+	+	abc	39	41	47

The design was blocked according to the following design table:

```
Factors:   3  Base Design:        3, 8
Runs:     24  Replicates:            3
Blocks:    3  Center pts (total):    0
Design Table

Run  Block  A  B  C
  1      1  -  -  -
  2      1  +  -  -
  3      1  -  +  -
  4      1  +  +  -
  5      1  -  -  +
  6      1  +  -  +
  7      1  -  +  +
  8      1  +  +  +
  9      2  -  -  -
 10      2  +  -  -
 11      2  -  +  -
 12      2  +  +  -
 13      2  -  -  +
 14      2  +  -  +
 15      2  -  +  +
 16      2  +  +  +
 17      3  -  -  -
 18      3  +  -  -
 19      3  -  +  -
 20      3  +  +  -
 21      3  -  -  +
 22      3  +  -  +
 23      3  -  +  +
 24      3  +  +  +
```

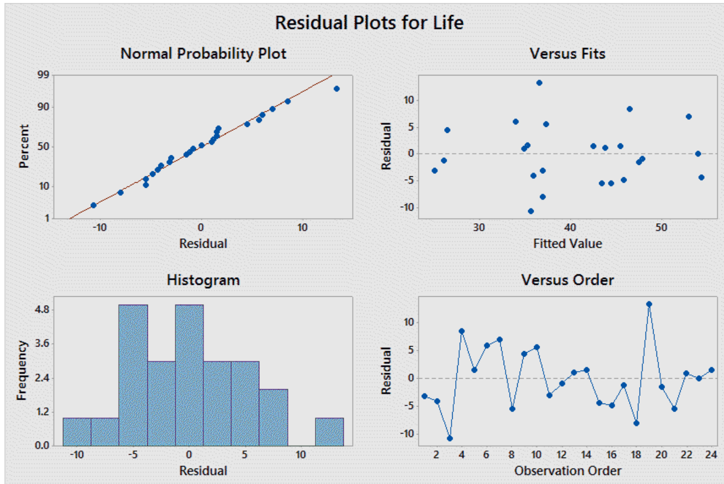
Our initial analysis of variance concluded that the significant effects were B, C, and AC, as is demonstrated in the normal probability plot of effects below.



Our model was then reanalyzed with all nonsignifcant factors lumped in with error, except for factor A which was preserved for hierarchy. The following Minitab Output represents this modified design.

```
Factorial Regression: Life versus Blocks, A, B, C

Analysis of Variance

Source                DF   Adj SS   Adj MS  F-Value  P-Value
Model                  6  1591.08  265.181     6.29    0.001
  Blocks               2     8.08    4.042     0.10    0.909
  Linear               3  1022.33  340.778     8.08    0.001
    A                  1     8.17    8.167     0.19    0.665
    B                  1   661.50  661.500    15.69    0.001
    C                  1   352.67  352.667     8.36    0.010
  2-Way Interactions   1   560.67  560.667    13.30    0.002
    A*C                1   560.67  560.667    13.30    0.002
Error                 17   716.75   42.162
Total                 23  2307.83


Model Summary

      S    R-sq  R-sq(adj)  R-sq(pred)
6.49321  68.94%     57.98%      38.10%
```

Again, B, C, and AC are found to be significant. This concurs with the original findings of Problem 6.1. Now, we must analyze the residuals to determine if there is any reason to question our assumptions.



Our normal probability plot give us no reason to question our normality assumption. The remaining residual plots indicate the possible presence of an outlier, which might bear further investigation. However, the effect isn't extreme and does not cause us to question our assumption of equal variances.

7.4 *Consider the data from the first replicate of Problem 6.1. Suppose that these observations could not all be run using the same bar stock. Set up a design to run these observations in two blocks of four observations each with ABC confounded. Analyze the data.*
The data for this experiment is reprinted in Problem 7.1 for reference.
The design was blocked according to the following design table with ABC confounded.

```
Full Factorial Design

Factors:  3   Base Design:        3, 8   Resolution with blocks:  IV
Runs:     8   Replicates:            1
Blocks:   2   Center pts (total):    0

Block Generators: ABC

Design Table

Run  Block  A  B  C
  1      1  -  -  -
  2      1  +  +  -
  3      1  +  -  +
  4      1  -  +  +
  5      2  +  -  -
  6      2  -  +  -
  7      2  -  -  +
  8      2  +  +  +
```

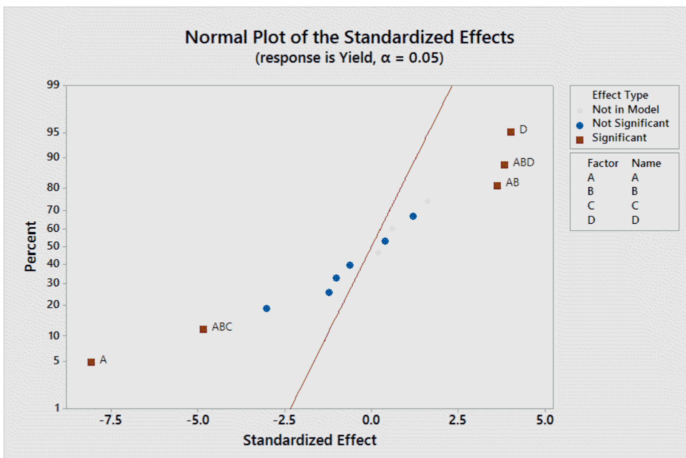
An initial analysis run with all factors reveals no apparent significant effects, but the percent contribution leads us to believe that factors B, C, and AC are significant. Therefore we will run the analysis again with the factors A, B, C, and AC (A is included for heirarchy) and the remaining factors will be lumped into the error term.
This results in the following analysis of variance.

Factorial Regression: Life versus Blocks, A, B, C

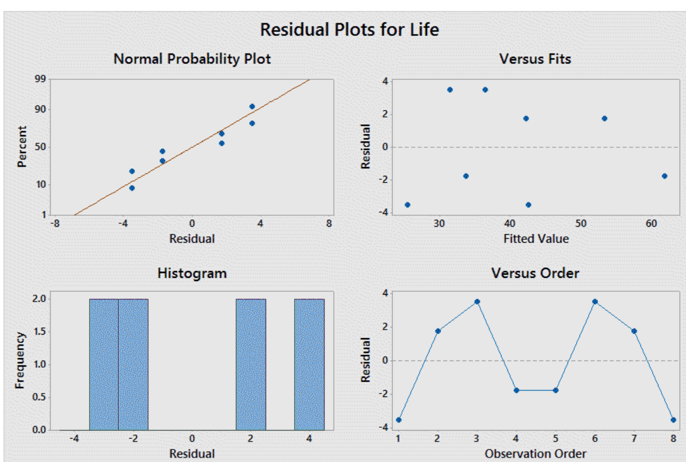
Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Model	5	987.63	94.16%	987.625	197.525	6.45	0.140
Blocks	1	91.13	8.69%	91.125	91.125	2.98	0.227
Linear	3	518.38	49.42%	518.375	172.792	5.64	0.154
A	1	3.12	0.30%	3.125	3.125	0.10	0.780
B	1	325.13	31.00%	325.125	325.125	10.62	0.083
C	1	190.13	18.13%	190.125	190.125	6.21	0.130
2-Way Interactions	1	378.13	36.05%	378.125	378.125	12.35	0.072
A*C	1	378.13	36.05%	378.125	378.125	12.35	0.072
Error	2	61.25	5.84%	61.250	30.625		
Total	7	1048.88	100.00%				

We see that the model is to sensitive to detect any significant factors at the 5% significance level, however at the 10% significance level factors B and AC are significant. This is illustrated by the normal probability plot for effects included below.



Now we will analyze the residuals to validate our model.



These residual plots are somewhat concerning. There seems to be a pattern in the normal probability plot. The histogram looks far from normal. The versus fits hints at a possible pattern, which is supported by the versus order graph. Based on these residual plots, we would recommend that a confirming experiment be conducted to test the veracity of our model.
Analytically, this is supported by the fact that factors B, C, and AC were all significant at the 5% level previously and factor C was not found to be significant at even the 10% level in this design.

7.5 *Consider the data from the first replicate of Problem 6.7. Construct a design with two blocks of eight observations each with ABCD confounded. Analyze the data.*

The experiment details and data are copied below for convienance:
6.7 *An experiment was performed to improve the yield of a chemical process Four factors were selected, and twoo replicates of a completely randomized experiment were run. The results are shown in the following table:*

Treatment Combination	Replicate I	Replicate II	Treatment Combination	Replicate I	Replicate II
(1)	90	93	d	98	95
a	74	78	ad	72	76
b	81	85	bd	87	83
ab	83	80	abd	85	86
c	77	78	cd	99	90
ac	81	80	acd	79	75
bc	88	82	bcd	87	84
abc	73	70	abcd	80	80

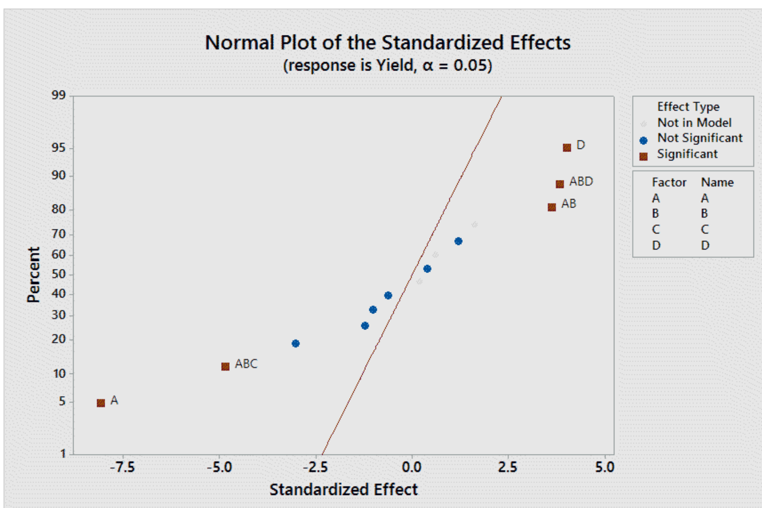
The design was blocked according to the following design table with ABCD confounded.

```
Factors:   4  Base Design:        4, 16  Resolution with blocks:  V
Runs:     16  Replicates:             1
Blocks:    2  Center pts (total):     0

Block Generators: ABCD

Design Table

Run  Block  A  B  C  D
  1      1  +  -  -  -
  2      1  -  +  -  -
  3      1  -  -  +  -
  4      1  +  +  +  -
  5      1  -  -  -  +
  6      1  +  +  -  +
  7      1  +  -  +  +
  8      1  -  +  +  +
  9      2  -  -  -  -
 10      2  +  +  -  -
 11      2  +  -  +  -
 12      2  -  +  +  -
 13      2  +  -  -  +
 14      2  -  +  -  +
 15      2  -  -  +  +
 16      2  +  +  +  +
```

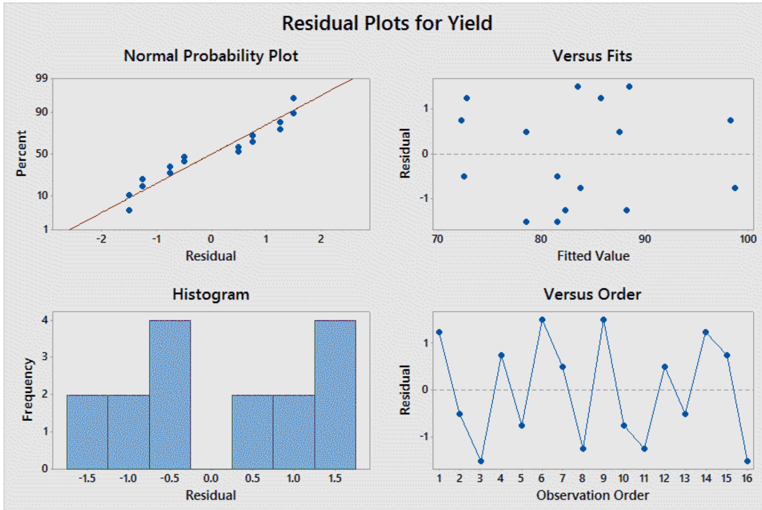
Our initial analysis of percent contribution leads us to believe that factors A, ABC, D, ABD, AB, and AD are possibly significant. Therefore, we will run the analysis of variance again with these factors. Factors B, AC, BC, and BD will also be included to preserve heirarchy.

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Model	12	941.250	98.07%	941.250	78.438	12.72	0.030
Blocks	1	42.250	4.40%	42.250	42.250	6.85	0.079
Linear	3	502.250	52.33%	502.250	167.417	27.15	0.011
A	1	400.000	41.68%	400.000	400.000	64.86	0.004
B	1	2.250	0.23%	2.250	2.250	0.36	0.588
D	1	100.000	10.42%	100.000	100.000	16.22	0.028
2-Way Interactions	6	162.500	16.93%	162.500	27.083	4.39	0.126
A*B	1	81.000	8.44%	81.000	81.000	13.14	0.036
A*C	1	1.000	0.10%	1.000	1.000	0.16	0.714
A*D	1	56.250	5.86%	56.250	56.250	9.12	0.057
B*C	1	6.250	0.65%	6.250	6.250	1.01	0.388
B*D	1	9.000	0.94%	9.000	9.000	1.46	0.314
C*D	1	9.000	0.94%	9.000	9.000	1.46	0.314
3-Way Interactions	2	234.250	24.41%	234.250	117.125	18.99	0.020
A*B*C	1	144.000	15.00%	144.000	144.000	23.35	0.017
A*B*D	1	90.250	9.40%	90.250	90.250	14.64	0.031
Error	3	18.500	1.93%	18.500	6.167		
Total	15	959.750	100.00%				

We see that at the $\alpha = 0.05\%$ significance level that factors AB, ABD, D, ABC, and A are significant, which concurs with our earlier assesment. This is also confirmed by the normal probability plot for effects below.



Now, we must analyze the residual plots to validate our model.



We see that there is a possible pattern in the normal probability plot the residuals which could indicate some curvature that is not correctly modeled by our design. This is supported by the histogram. The versus fits and versus order do not give us any immediate cause for concern. We would recommend performing a confirmation experiment to further validate the model.

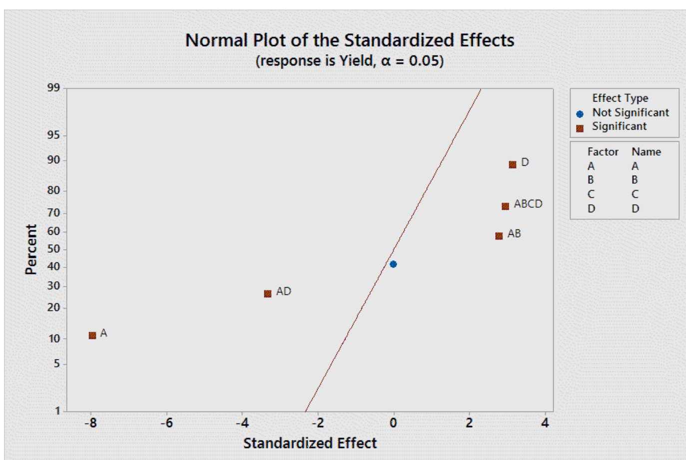
7.6 *Repeat Problem 7.5 assuming that four blocks are required. Confound ABD and ABC (and, consequently, CD) with blocks.*
The design was blocked according to the following design table with ABD and ABC confounded.

```
Factors:   4   Base Design:         4, 16   Resolution with blocks:  III
Runs:     16   Replicates:              1
Blocks:    4   Center pts (total):      0

* NOTE * Blocks are confounded with two-way interactions.
Block Generators: ABD, ABC

Design Table

Run  Block  A  B  C  D
  1      1  -  -  -  -
  2      1  +  +  -  -
  3      1  +  -  +  +
  4      1  -  +  +  +
  5      2  +  -  +  -
  6      2  -  +  +  -
  7      2  -  -  -  +
  8      2  +  +  -  +
  9      3  -  -  +  -
 10      3  +  +  +  -
 11      3  +  -  -  +
 12      3  -  +  -  +
 13      4  +  -  -  -
 14      4  -  +  -  -
 15      4  -  -  +  +
 16      4  +  +  +  +
```

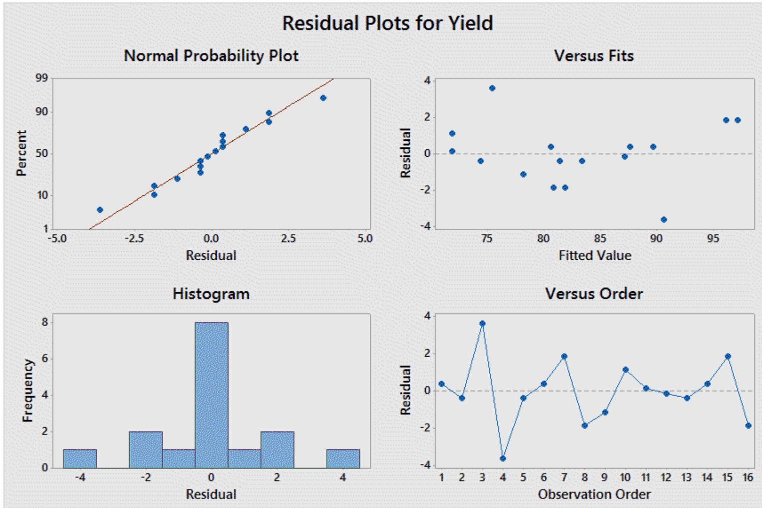
Our initial analysis of percent contribution leads us to believe that factors A, D, AB, AD, and ABCD are possibly significant. Therefore, we will run the analysis of variance again with these factors. Factor C will also be included to preserve heirarchy.

```
Analysis of Variance

Source                DF   Seq SS  Contribution   Adj SS   Adj MS  F-Value  P-Value
Model                  9  930.250        95.51%  930.250  103.361    14.18    0.002
  Blocks               3  194.500        19.97%  194.500   64.833     8.89    0.013
  Linear               3  534.500        54.88%  534.500  178.167    24.43    0.001
    A                  1  462.250        47.46%  462.250  462.250    63.39    0.000
    C                  1    0.000         0.00%    0.000    0.000     0.00    1.000
    D                  1   72.250         7.42%   72.250   72.250     9.91    0.020
  2-Way Interactions   2  137.250        14.09%  137.250   68.625     9.41    0.014
    A*B                1   56.250         5.78%   56.250   56.250     7.71    0.032
    A*D                1   81.000         8.32%   81.000   81.000    11.11    0.016
  4-Way Interactions   1   64.000         6.57%   64.000   64.000     8.78    0.025
    A*B*C*D            1   64.000         6.57%   64.000   64.000     8.78    0.025
Error                  6   43.750         4.49%   43.750    7.292
Total                 15  974.000       100.00%
```

We see that at the $\alpha = 0.05\%$ significance level that factors A, D, AB,AD, and ABCD are significant, which concurs with our earlier assesment. This particularly interesting since the data is the same as the in Problem 7.5. Yet different factors were found to be significant. This illustrated just how important blocking is in experimental design. The significance of the factors is confirmed below in the normal probabilty plot for the effects.



Now we must analyze the residual plots to validate our model.



There is nothing in our residual plots to make us question the model. Considering that these residual plots are 'better' than the residual plots of the previous problem (where the data was blocked two ways), we might conclude that the blocking four ways better models the situation. This can be confirmed using a confirmation experiment.

7.16 *Consider the 2^6 design in eight blocks of eight runs each with ABCD, ACE, and ABEF as the independent effects chosen to be confounded with blocks. Generate the design. Find the other effects confounded with blocks.*

The design is blocked eight ways with ABCD, ACE, and ABEF as the confounded independent effects.

```
Block Generators: ABCD, ACE, ABEF
Design Table
```

Run	Block	A	B	C	D	E	F
1	1	−	+	−	−	−	−
2	1	+	−	+	+	−	−
3	1	−	−	+	−	+	−
4	1	+	+	−	+	+	−
5	1	+	+	+	−	−	+
6	1	−	−	−	+	−	+
7	1	+	−	−	−	+	+
8	1	−	+	+	+	+	+
9	2	+	−	+	−	−	−
10	2	−	+	−	+	−	−
11	2	+	+	−	−	+	−
12	2	−	−	+	+	+	−
13	2	−	−	−	−	−	+
14	2	+	+	+	+	−	+
15	2	−	+	+	−	+	+
16	2	+	−	−	+	+	+
17	3	+	−	−	−	−	−
18	3	−	+	+	+	−	−
19	3	+	+	+	−	+	−
20	3	−	−	−	+	+	−
21	3	−	−	+	−	−	+
22	3	+	+	−	+	−	+
23	3	−	+	−	−	+	+
24	3	+	−	+	+	+	+
25	4	−	+	+	−	−	−
26	4	+	−	−	+	−	−
27	4	−	−	−	−	+	−
28	4	+	+	+	+	+	−
29	4	+	+	−	−	−	+
30	4	−	−	+	+	−	+
31	4	+	−	+	−	+	+
32	4	−	+	−	+	+	+
33	5	+	+	+	−	−	−
34	5	−	−	−	+	−	−
35	5	+	−	−	−	+	−
36	5	−	+	+	+	+	−
37	5	−	+	−	−	−	+
38	5	+	−	+	+	−	+
39	5	−	−	+	−	+	+
40	5	+	+	−	+	+	+
41	6	−	−	−	−	−	−
42	6	+	+	+	+	−	−
43	6	−	+	+	−	+	−
44	6	+	−	−	+	+	−
45	6	+	−	+	−	−	+
46	6	−	+	−	+	−	+
47	6	+	+	−	−	+	+
48	6	−	−	+	+	+	+
49	7	−	−	+	−	−	−
50	7	+	+	−	+	−	−
51	7	−	+	−	−	+	−
52	7	+	−	+	+	+	−
53	7	+	−	−	−	−	+
54	7	−	+	+	+	−	+
55	7	+	+	+	−	+	+
56	7	−	−	−	+	+	+
57	8	+	+	−	−	−	−
58	8	−	−	+	+	−	−
59	8	+	−	+	−	+	−
60	8	−	+	−	+	+	−
61	8	−	+	+	−	−	+
62	8	+	−	−	+	−	+
63	8	−	−	−	−	+	+
64	8	+	+	+	+	+	+

We see from the following excerpt of the aliases that the other effects that are confounded with blocks are BDE, CDEF, BCF, ADF.

```
Blk1 = ABCD
Blk2 = ACE
Blk3 = ABEF
Blk4 = BDE
Blk5 = CDEF
Blk6 = BCF
Blk7 = ADF
```

7.18 *Consider the data in Example 7.2. Suppose that all of the observations in block 2 are increased by 20. Analyze the data that would result. Estimate the block effect. Can you explain its magnitude? Do the blocks now appear to be an important factor? Are there any other effect estimates impacted by the changes that you made to the data?*
The design was blocked according to the following design table with ABCD confounded.

Design Table

Run	Block	A	B	C	D
1	1	+	-	-	-
2	1	-	+	-	-
3	1	-	-	+	-
4	1	+	+	+	-
5	1	-	-	-	+
6	1	+	+	-	+
7	1	+	-	+	+
8	1	-	+	+	+
9	2	-	-	-	-
10	2	+	+	-	-
11	2	+	-	+	-
12	2	-	+	+	-
13	2	+	-	-	+
14	2	-	+	-	+
15	2	-	-	+	+
16	2	+	+	+	+

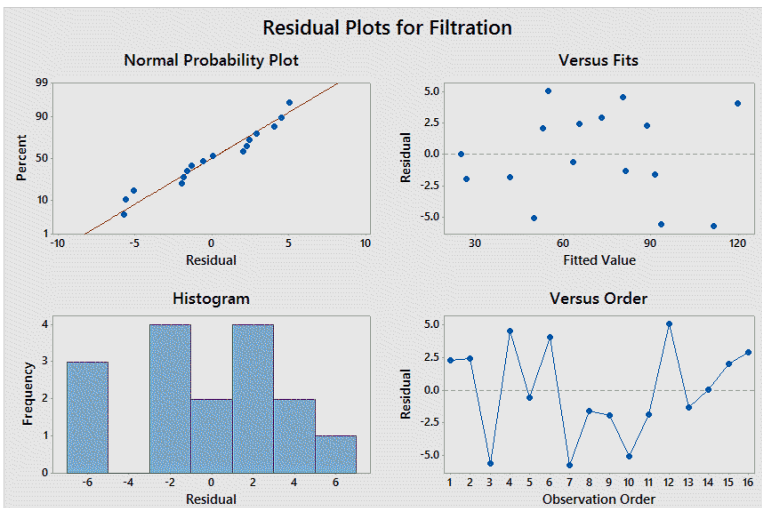
Our initial analysis of percent contribution leads us to believe that factors A, C, D, AC, and AD are possibly significant. Therefore we will run the analysis of variance again with these factors.

Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Model	6	11503.4	98.40%	11503.4	1917.23	92.00	0.000
Blocks	1	5967.6	51.04%	5967.6	5967.56	286.35	0.000
Linear	3	3116.2	26.65%	3116.2	1038.73	49.84	0.000
A	1	1870.6	16.00%	1870.6	1870.56	89.76	0.000
C	1	390.1	3.34%	390.1	390.06	18.72	0.002
D	1	855.6	7.32%	855.6	855.56	41.05	0.000
2-Way Interactions	2	2419.6	20.70%	2419.6	1209.81	58.05	0.000
A*C	1	1314.1	11.24%	1314.1	1314.06	63.05	0.000
A*D	1	1105.6	9.46%	1105.6	1105.56	53.05	0.000
Error	9	187.6	1.60%	187.6	20.84		
Total	15	11690.9	100.00%				

We see that at the $\alpha = 0.05\%$ significance level that factors A, C, D, AC, and AD are significant, which concurs with our earlier assement. This is also confirmed by the normal probability plot for effects below.

Now, we must analyze the residual plots to validate our model.



We see that there is possible pattern in the normal probability plot for the residuals which may indicate that we are not properly modeling curvature. However, the versus fits and versus order plots do not show any significant patterns, which may indicate that we have properly accounted for stochastic randomization. It might be wise to perform a confirming experiment.

In order to estimate the block effect, we will simply do as follows

$$\textbf{Block Effect } = \bar{y}_{Block1} - \bar{y}_{Block2} = \frac{715}{8} - \frac{406}{8} = 38.625$$

This is the inverse of the block effect estimated in Example 7.2 with the additional 20 units that were added to block 2. It is the inverse because the blocks changed designation when putting it into Minitab, however the magnitude is the same. All other effects remained the same, as is seen in the above ANOVA.Therefore, blocks do not appear to be an important factor as they were correctly accounted for.