CSCI 628
Lab 2

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Rocket Control

You have been hired to improve the efficiency of a rocket that needs to fly in a straight path from position 0 to position N. The fuel consumption of the rocket is a function of the acceleration/deceleration. We model the fuel consumption and the rockets speed and position, by discretizing time, and letting x_t*,* v_t *and* a_t *be the position, velocity, and acceleration (where* $a_t < 0$ *means a deceleration), respectively, of the rocket at time t. The rocket starts at position 0 at speed 0, so*

$$x_0 = 0$$
$$v_0 = 0$$

It needs to land at position N at time T, and we wish the rocket to land softly, so

$$x_T = N$$
$$v_T = 0$$

a.) Give linear constraints that govern the position and speed of the rocket at every time step.

Discretizing the model gives us the following constraints

$$x_{t+1} = x_t + v_t \left(x_t = x_0 + \sum_{i=0}^{T-1} v_i \right)$$
$$v_{t+1} = v_t + a_t \left(v_t = v_0 + \sum_{i=0}^{T-1} a_i \right)$$

b.) Suppose that instead of fuel consumption, we want to minimize the maximum thrust required, i.e., $\max_{t=0,\ldots,T-1} |a_t|$. *Formulate the problem of minimizing the maximum thrust by a linear program and implement it in AMPL.*

minimize z
subject to $z \geq -a_t, t = 0, \ldots, T-1$
$z \geq a_t, t = 0, \ldots, T-1$
$x_{t+1} = x_t + v_t, t = 0, \ldots, T-1$
$v_{t+1} = v_t + a_t t = 0, \ldots, T-1$
$X_T = N$
$v_T = 0$
$x_0 = 0$
$v_0 = 0$

Suppose the total fuel consumption of the rocket is given by $\sum_{t=0}^{T-1} |a_t|$. *Formulate the problem of minimizing the fuel consumption by a linear program and implement it in AMPL.*

Option 1: Add variable z_t for $t = 0, \ldots, T-1$ and add constraints $z_t \geq a_t$ and $z_t \geq -a_t$, $t = 0, \ldots, T-1$.

Option 2: Have two variables: a_t^+, a_t^- for every $t = 0, \ldots, T-1$. Also add constraints that $a_t = a_t^+ - a_t^-$. Write objective as minimize $\sum_{t=0}^{T-1} (a_t^+ + a_t^-)$.

minimize $\sum_{t=0}^{T-1} (a_t^+ + a_t^-)$
subject to $a_t = a_t^+ - a_t^-$
$x_{t+1} = x_t + v_t, t = 1, \ldots, T-1$
$v_{t+1} = v_t + a_t t = 1, \ldots, T-1$
$X_T = N$
$v_T = 0$
$x_0 = 0$

$v_0 = 0$

SailCO Variations

If you have time left, try the following modifications for the SailCO problem. You can use your own SailCO.mod, or download SailCO.mod from blackboard, which contains the integer program for the following production planning problem (Extra Credit problem from Lab 1):

- projected demand:

	Q1	Q2	Q3	Q4
demand	40	60	75	25

- production cost:
  - $400/boat, first 40 boats in a quarter
  - $450/boat, each additional boat
- inventory cos:
  - $20/boat/quarter for boats on hand at the end of a quarter after production has occurred and demand satisfied
  - initial inventory: 10 sailboats for the start of Q1

Production Smoothing

- Modify your model so that the inventory at the end of Q4 must be 10. In addition to the costs already in the model, there is a cost associated with increasing/decreasing the production. For each unit of increase in production from one quarter to the next, there is a cost of $400/boat (cost of hiring and training new employee) and for each unit of decrease in production there is a cost of $500/boat (severence pay, decreasing morale, etc.)
- Assume that the production in the quarter before Q1 was 50, the inventory at the start of Q1 is 10.

Allowing backlogged demand

- Suppose now that demands can be backlogged and met in future periods.
- Penalty of $100/boat for each quarter that demand is unmet.
- Must meet all demand by end of Q4 (and have an inventory of 10 boats left, as in the previous variation.