Formalizing Mathematical Developments to Support Verifying Compilers

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A Grand Challenge

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- $P = NP$
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- Fermat’s Last Theorem
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The growth of large-scale software engineering will increase the costliness and frequency of these errors.
What are some traditional software testing methods?

- Debugging by individual software engineers.
- Beta testing by interns (often unpaid).
- Specialized commercial testing software.

Traditional methods are only able to test code under a subset of possible conditions and a subset of possible inputs. They cannot guarantee the absence of errors.
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Verifying compilers prove with mathematical certainty the absence of errors in the generated code.
Verifying Compiler Overview

- Program Code & Specifications
- Mathematical Theories
- Verification Conditions Generator
- Automated Prover
- Proof Results

Verifying Compiler
Clemson RSRG

Clemson RESOLVE Software Research Group is developing a push-button verifying compiler [1].

Definition

A push-button verifying compiler generates mathematical proofs of correctness and executable code in the same way regular compilers generate code [3].

An integrated software language called RESOLVE is being developed to support the compiler.
Clemson RESOLVE Software Research Group is developing a push-button verifying compiler [1].

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The automated prover operates on a mathematical framework which relies on theory files stored in a coded math library.
The RESOLVE push-button verifying compiler utilizes a free web-based integrated development environment.
Principal areas for the primary math library:
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- Integer Theory
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- Integer Theory
- Natural Number Theory
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With select theories, definitions, and properties from these areas, the automated prover is able to prove generated verification conditions.
The math theory library is integrated directly into the RESOLVE web IDE.
RESOLVE Code determining the maximum of two integers:

```plaintext
Facility Int_Max_Example_Facility;
Operation Max(restores I: Integer; restores J: Integer) : Integer;
ensures (Max = I or Max = J) and (Max >= I and Max >= J);
Procedure
Max := I + J;
If (I > J) then
Max := Max - J;
end;
If (J > I) then
Max := Max - I;
end;
end Max;
end Int_Max_Example_Facility;
```
Verification Condition Example

Example of a verification condition for the maximum integer program:

Goal:

\(((I + J) - J) - I) = I \text{ or } (((I + J) - J) - I) = J\)

Given:

\(\text{min\_int} \leq 0\)
\(0 < \text{max\_int}\)
\(\text{Last\_Char\_Num} > 0\)
\(\text{min\_int} \leq J \text{ and } J \leq \text{max\_int}\)
\(\text{min\_int} \leq I \text{ and } I \leq \text{max\_int}\)
\(I > J\)
\(J > I\)
Excerpt from the basic binary operations theory file:

**Precis Basic_Binary_OPERATION_Properties;**
uses Boolean_Theory;

Definition **Is_Associative**(omicron : (D : SSet) * D -> D): B =
For all x, y, z : D,
omicron(x, omicron(y,z)) = omicron(omicron(x, y), z);

Definition **Is_Commutative**(omicron : (D : SSet) * D -> D, x : D) : B =
Is_Commutator_for(omicron, x);

Theorem I7: **Is_Associative**(+);

Theorem I10: **Is_Commutative**(+);
RESOLVE Translator: C

RESOLVE generated verified executable code for maximum integer program in C:

```c
int Max(int I, int J){int Max= 0;
    /*ensuresMaxIMaxJMaxIMaxJ*/
    Max = I + J;
    if(I > J){
        Max = Max - J;
    }
    if(J > I){
        Max = Max - I;
    }
    return Max; }
```
RESOLVE Translator: Java

RESOLVE generated verified executable code for maximum integer program in Java:

```java
public static class Int_Max_Example_Facility{

public static int Max(int I, int J){int Max= 0;
/*ensures*/

Max = I + J;
if(I > J){
Max = Max - J;
}
if(J > I){
Max = Max - I;}
return Max; } }
```
RESOLVE code building the integers:

Precis Basic_Integer_Theory;
uses Monogenerator_Theory, Basic_Function_Properties,
Basic_Ordering_Theory, Basic_Natural_Number_Theory;

Categorical Definition introduces Z: SSet, 0 : Z, NB : Z -> Z
related by (Is_Monogeneric_for(Z, 0, NB));

Definition 1 : Z = (suc(0));
Corollary 1: For all m : Z, suc(m) = m + 1;
Corollary 2: 1 : NN;
Corollary 3: 4 not(=) 0;

Theorem I15: For all m : Z, For all n : Z, -(m * n) = (-m) * n;
Theorem I15: For all m : Z, For all n : Z, m * (-n) = (-m * n);
RESOLVE code building the Natural Numbers using successor property:

Precis Basic_Natural_Number_Theory;
uses Basic_Binary_Operation_Properties,Basic_Ordering_Theory;

Categorical Definition introduces $N : SSet$, $0 : N$, $suc : N \to N$
related by $(\text{Is\_Monogeneric\_for}(N,0,suc))$;

Definition 2: $N = (\text{suc}(1))$;
Definition 3: $N = (\text{suc}(2))$;
Definition 4: $N = (\text{suc}(3))$;
Definition 5: $N = (\text{suc}(4))$;
Definition 6: $N = (\text{suc}(5))$;
Definition 7: $N = (\text{suc}(6))$;
Definition 8: $N = (\text{suc}(7))$;
Definition 9: $N = (\text{suc}(8))$;
Fundamental Theorems

In addition to basic properties and definitions, RESOLVE can code more complex theorems, including some famous examples...

Precis Major_Theorems;
uses Boolean_Theory, Set_Theory, Basic_Natural_Number_Theory;

Theorem Well_Ordering_Principle:
For all D : SSet,
D /= empty_set implies
(There exists min_element : D such that
(For all x : D, min_element <= x ));

Theorem Archimedean_Property:
(x : R and y: R and x >0)
implies (There exists n : N such that n > 0 and n*x > y );
My immediate goal is to finalize the development of a math library which includes a full complement of basic theories and definitions.
Areas of Future Research

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The end goal is the full development of a successful verifying compiler and a resolution to the grand challenge.
References

[1] Resolve website
