Formalizing Mathematical Developments to Support Verifying Compilers

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I would like to thank my mentor, Dr. Murali Sitaraman from Clemson University for his continued support and guidance.

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Outline

1. Verifying Compilers
2. Clemson RSRG
3. Math Theory Library
4. Areas of Future Research

A Grand Challenge

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- Fermat's Last Theorem

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The growth of large-scale software engineering will increase the costliness and frequency of these errors.

Traditional Testing Methods

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Traditional methods are only able to test code under a subset of possible conditions and a subset of possible inputs.

They cannot guarantee the absence of errors.

Verifying Compilers

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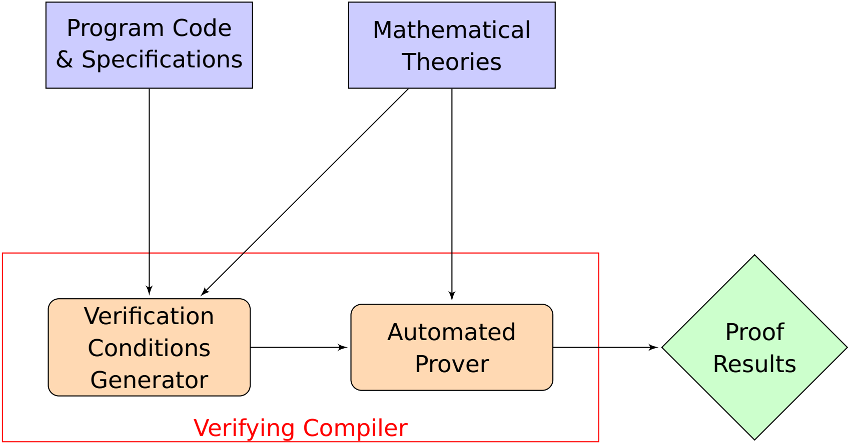
Verifying compilers prove with mathematical certainty the absence of errors in the generated code.

Verifying Compiler Overview



Program Code & Specifications

Mathematical Theories

Verification Conditions Generator

Automated Prover

Proof Results

Verifying Compiler

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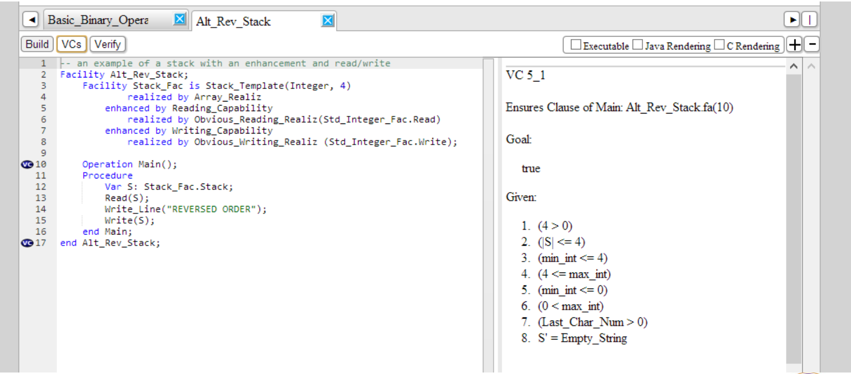
The automated prover operates on a mathematical framework which relies on theory files stored in a coded math library.

RESOLVE Web IDE

The RESOLVE push-button verifying compiler utilizes a free web-based integrated development enviroment.



```
-- an example of a stack with an enhancement and read/write
Facility Alt_Rev_Stack;
    Facility Stack_Fac is Stack_Template(Integer, 4)
            realized by Array_Realiz
        enhanced by Reading_Capability
            realized by Obvious_Reading_Realiz(Std_Integer_Fac.Read)
        enhanced by Writing_Capability
            realized by Obvious_Writing_Realiz (Std_Integer_Fac.Write);

    Operation Main();
    Procedure
        Var S: Stack_Fac.Stack;
        Read(S);
        Write_Line("REVERSED ORDER");
        Write(S);
    end Main;
end Alt_Rev_Stack;
```

```
VC 5_1

Ensures Clause of Main: Alt_Rev_Stack.fa(10)

Goal:

    true

Given:

    1. (4 > 0)
    2. (|S| <= 4)
    3. (min_int <= 4)
    4. (4 <= max_int)
    5. (min_int <= 0)
    6. (0 < max_int)
    7. (Last_Char_Num > 0)
    8. S' = Empty_String
```

Mathematical Developments

Principal areas for the primary math library:

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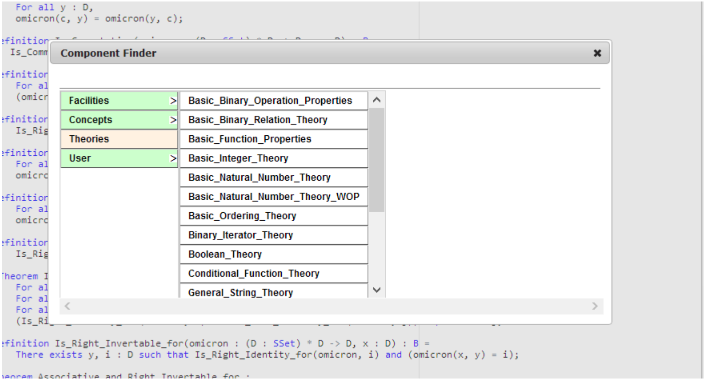
With select theories, definitions, and properties from these areas, the automated prover is able to prove generated verification conditions.

Math Theory Library

The math theory library is integrated directly into the RESOLVE web IDE.



```
   For all y : D,
   omicron(c, y) = omicron(y, c);

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   omicrc

efinitior
   Is_Rig

Theorem I
   For al
   For al
   For al
   (Is_Ri

efinition Is_Right_Invertable_for(omicron : (D : SSet) * D -> D, x : D) : B =
   There exists y, i : D such that Is_Right_Identity_for(omicron, i) and (omicron(x, y) = i);

eorem Associative and Right Invertable for :
```

RESOLVE Code

RESOLVE Code determining the maximum of two integers:

```
Facility Int_Max_Example_Facility;
Operation Max(restores I: Integer; restores J: Integer) : Integer;
ensures (Max = I or Max = J) and (Max >= I and Max >= J);
Procedure
Max := I + J;
If (I > J) then
Max := Max - J;
end;
If (J > I) then
Max := Max - I;
end;
end Max;
end Int_Max_Example_Facility;
```

Verification Condition Example

Example of a verification condition for the maximum integer program:

```
Goal:
((((I + J) - J) - I) = I or (((I + J) - J) - I) = J)

Given:
(min_int <= 0)
(0 < max_int)
(Last_Char_Num > 0)
(min_int <= J) and (J <= max_int)
(min_int <= I) and (I <= max_int)
(I > J)
(J > I)
```

Example Theory File

Excerpt from the basic binary operations theory file:

```
Precis Basic_Binary_Operation_Properties;
uses Boolean_Theory;

Definition Is_Associative(omicron : (D : SSet) ∗ D -> D): B =
For all x, y, z : D,
omicron(x, omicron(y,z)) = omicron(omicron(x, y), z);

Definition Is_Commutative(omicron : (D : SSet) ∗ D -> D, x : D) : B =
Is_Commutator_for(omicron, x);

Theorem I7: Is_Associative(+);

Theorem I10: Is_Commutative(+);
```

RESOLVE Translator: C

RESOLVE generated verified executable code for maximum integer program in C:

```
int Max(int  I, int  J){int Max= 0;
/*ensuresMaxIMaxJMaxIMaxJ*/

Max = I + J;
if(I > J){
Max = Max - J;
}
if(J > I){
Max = Max - I;}return Max; }
```

RESOLVE Translator: Java

RESOLVE generated verified executable code for maximum integer program in Java:

```
public static class Int_Max_Example_Facility{

public static int Max(int  I, int  J){int Max= 0;
/*ensures*/

Max = I + J;
if(I > J){
Max = Max - J;
}
if(J > I){
Max = Max - I;}return Max; } }
```

Integer Theory

RESOLVE code building the integers:

```
Precis Basic_Integer_Theory;
uses Monogenerator_Theory, Basic_Function_Properties,
Basic_Ordering_Theory, Basic_Natural_Number_Theory;

   Categorical Definition introduces Z: SSet,  0 : Z, NB : Z -> Z
    related by (Is_Monogeneric_for(Z, 0, NB));

    Definition 1 : Z = (suc(0));
    Corollary 1: For all m : Z, suc(m) = m + 1;
    Corollary 2: 1 : NN;
    Corollary 3: 4 not(=) 0;

    Theorem I15: For all m : Z, For all n : Z, -(m * n) = (-m) * n;
    Theorem I15: For all m : Z, For all n : Z, m * (-n) = (-m * n);
```

Natural Number Theory

RESOLVE code building the Natural Numbers using successor property:

```
Precis Basic_Natural_Number_Theory;
uses Basic_Binary_Operation_Properties,Basic_Ordering_Theory;

    Categorical Definition introduces N : SSet, 0 : N, suc : N -> N
    related by (Is_Monogeneric_for(N,0,suc));

    Definition 2: N = (suc(1));
    Definition 3: N = (suc(2));
    Definition 4: N = (suc(3));
    Definition 5: N = (suc(4));
    Definition 6: N = (suc(5));
    Definition 7: N = (suc(6));
    Definition 8: N = (suc(7));
    Definition 9: N = (suc(8));
```

Fundamental Theorems

In addition to basic properties and definitions, RESOLVE can code more complex theorems, including some famous examples⋯

```
Precis Major_Theorems;
uses Boolean_Theory, Set_Theory, Basic_Natural_Number_Theory;

Theorem Well_Ordering_Principle:
For all D : SSet,
D /= empty_set implies
(There exists min_element : D such that
(For all x : D, min_element <= x ));

Theorem Archimedean_Property:
(x : R and y: R and x >0)
implies (There exists n : N such that n > 0 and n*x > y );
```

Areas of Future Research

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The end goal is the full development of a successful verifying compiler and a resolution to the grand challenge.

References

[1] Resolve website http://http://www.cs.clemson.edu/resolve/.

[2] T. Hoare. The verifying compiler: A grand challenge for computing research. *J. ACM*, 50(1):63–69, Jan. 2003.

[3] M. Sitaraman, B. Adcock, J. Avigad, D. Bronish, P. Bucci, D. Frazier, H. M. Friedman, H. Harton, W. Heym, J. Kirschenbaum, J. Krone, H. Smith, and B. W. Weide. Building a push-button resolve verifier: Progress and challenges. *Form. Asp. Comput.*, 23(5):607–626, Sept. 2011.